Moldova State University
Faculty of Mathematics and Computer Science
Vladimir Andrunachievici Institute of Mathematics and Computer Science

INTERNATIONAL CONFERENCE

MATHEMATICS & INFORMATION TECHNOLOGIES: RESEARCH AND EDUCATION

(MITRE-2025)

Mathematical Week in Chisinau
dedicated to the centenary of Valentin Belousov (1925-1988)

ABSTRACTS

Chişinău, 2025

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INTERNATIONAL CONFERENCE MITRE-2025

The International Conference "Mathematics & IT: Research and Education (MITRE-2025)" is organized by the Faculty of Mathematics and Computer Science, the Center for Education and Research in Mathematics and Computer Science (CECMI) of the Moldova State University (**June 26 – 29, 2025**, Chişinău, Republic of Moldova).

The Conference MITRE in 2025 is at its 10th edition.

The MITRE-2025 Conference is organized alongside the **Quasigroups and Related Systems Conference** as part of the **Mathematics Week** in Chişinău (June 26 – July 3, 2025), dedicated to the centenary of the birth of the mathematician **Valentin Belousov** (1925–1988).

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1. Algebra, Geometry and Topology
2. Analysis, Differential Equations and Dynamical Systems
3. Applied Mathematics
4. Computer Science and Information Technologies
5. Didactics of Mathematics and Informatics

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This book contains the abstracts of communications, presented at the Conference MITRE-2025 (some communications in the Section Didactics of Mathematics and Informatics are presented in Romanian). The authors are responsible for the content of their abstract. The authors are listed in the alphabetical order by the last name of the first author within the conference sections.

The Organizing Committee MITRE-2025 thanks the authors for contribution with their abstracts.

PLENARY LECTURES

ATOMIC CARDINALITIES

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In mathematics, set theories with atoms were originally described in order to prove the independence of the axiom of choice from the other axioms of set theory. Later, they were used to formally model concepts in programming such as renaming, freshness and binding, as well as for providing a finitary representation of infinite sets that contain symmetries. Zermelo's original (1908) axiomatization of set theory with atoms (ZFA) allows objects (atoms) which may be members of sets, but are not made up of other elements. Finitely supported sets [1] are related to the permutation models of ZFA. A finitely supported set is a set equipped with a group action of the group of all finite permutations of the infinite set of atoms satisfying a finite support requirement; this requirement states that for any element x there exists a finite set of atoms S such that any permutation of atoms fixing S pointwise also leaves the element x invariant under the related group action.

On the class of finitely supported sets, one may define the relation ρ by stating that $X \rho Y$ if and only if there is a finitely supported bijective function $f : X \to Y$. We can prove that the binary relation ρ is a finitely supported equivalence relation. The equivalence class of a finitely supported set X with respect to ρ is called the *atomic cardinality* of X and it is denoted by $card(X)$. We can define the following arithmetic operations on atomic cardinalities: $card(X) + card(Y) = card(X + Y)$, where $X + Y = \{(x, 0) \,|\, x \in X\} \cup \{(y, 1) \,|\, y \in Y\}$; $card(X) \cdot card(Y) = card(X \times Y)$ and $card(X)^{card(Y)} = card(\{f : Y \to X \,|\, f \text{ is finitely supported}\})$.

We are able to prove that atomic cardinalities have similar properties as in Zermelo-Fraenkel set theory: $card(Z)^{card(X) \cdot card(Y)} = (card(Z)^{card(X)})^{card(Y)}$, $card(Z)^{card(X)+card(Y)} = card(Z)^{card(X)} \cdot card(Z)^{card(Y)}$, $card(\mathcal{P}(X)) = 2^{card(X)}$ $(card(X) \cdot card(Y))^{card(Z)} = card(X)^{card(Z)} \cdot card(Y)^{card(Z)}$.

On the family of atomic cardinalities we may introduce two binary relations ρ_1 by: $card(X) \rho_1 card(Y)$ if and only if there exists a finitely supported injective function $f : X \to Y$, and ρ_2 by: $card(X) \rho_2 card(Y)$ if and only if there is a finitely supported surjective function $f : Y \to X$.

[1] *Speaking author*: A. Alexandru

Theorem 1.

- The binary relation ρ_1 is finitely supported, reflexive, anti-symmetric and transitive. However, the relation ρ_1 is not total.
- The binary relation ρ_2 is finitely supported, reflexive and transitive. However, the relation ρ_2 is not anti-symmetric, neither total.

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QUADRATIC QUASIGROUPS, NEARFIELDS AND ASSOCIATIVE TRIPLES

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By a quasigroup $(Q, *)$ we understand a binary operation such that $a * x = b$ and $y * a = b$ have unique solutions x and y for any choice of $a, b \in Q$. Call Q *maximally nonassociative* if $x * (y * z) = (x * y) * z$ implies $x = y = z$. It may be proved that if Q is maximally nonassociative, then $*$ is idempotent, i.e., $x * x = x$ for each $x \in Q$. The existence of finite maximally nonassociative quasigroups was in doubt for decades.

Finite (left) nearfields are equivalent to finite sharply doubly transitive permutation groups. They may be described by removing the right distributive law from axioms of a field. The best known proper nearfields are the quadratic nearfields of Dickson defined on $\mathbb{F}_{q^2}$, q a power of an odd prime, with multiplication

$$x \circ y = \begin{cases} xy & \text{if } y \text{ a square} \\ xy^q & \text{if } y \text{ a nonsquare} \end{cases} .$$

If $(N, +, \circ, 0, 1)$ is a nearfield, then for each $c \in N$ distinct from 1 there may be defined a quasigroup operation $x *_c y = x + (y - x)c$ (if N is a field this is the operation of the weighted mean). Each mapping $x \mapsto a \circ x + b$ is an automorphism of $(N, *_c)$. In some cases $(N, *_c)$ is maximally nonassociative. In fact, computer experiments show that such a c might exist for each finite N. A rigorous proof exists, up to now, only for quadratic nearfields.

This method cannot produce a maximally nonassociative quasigroup of prime order. However, the construction of a quasigroup from a nearfield may be mimicked by defining an operation $*_{a,b}$ on $\mathbb{F}_q$ determined by parameters $a, b \in \mathbb{F}_q$ as follows:

$$x *_{a,b} y = \begin{cases} x + a(y-x) & \text{if } y-x \text{ a square} \\ x + b(y-x) & \text{if } y-x \text{ a nonsquare} \end{cases}.$$

This operation is a quasigroup if and only if both ab and $(a-1)(b-1)$ are nonzero squares. Such quasigroups are called *quadratic*.

It turns out that in many cases $(\mathbb{F}_q, *_{a,b})$ is a maximally nonassociative quasigroup.

In the talk I will report for which orders a maximally nonassociative quasigroup is known to exist. Also I will discuss what is the probability that $(N, *_c)$ or $\mathbb{F}_q(*_{a,b})$ is maximally nonassociative. Furthermore, we shall see when two quadratic quasigroups are isomorphic.

Results mentioned above may be found in the following papers [1-4]:

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POTENTIAL SYSTEMS WITH RELATIVISTIC TYPE OPERATOR

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We are concerned with solvability of a nonlinear system involving a singular ϕ-Laplacian operator

$$u \mapsto [\phi(u')]'$$

associated with a potential multivalued boundary condition expressed in terms of subdifferential of a convex function. Here, ϕ is a homeomorphism from an open ball of radius a, centered at the origin B_a onto $\mathbb{R}^N$ such that $\phi(0_{\mathbb{R}^N}) = 0_{\mathbb{R}^N}$, $\phi = \nabla\Phi$, with $\Phi : \overline{B}_a \to (-\infty, 0]$ of class C^1 on B_a, continuous and strictly convex on $\overline{B}_a$. The prototype of such a ϕ-Laplacian is the *relativistic operator*, corresponding to $\phi(y) = y/\sqrt{1-|y|^2}$, $y \in B_1$.

First, we provide a variational approach of the system in the frame of critical point theory for convex, lower semicontinuous perturbations of C^1-functionals developed in [4]. Then we derive the existence of solutions either as minimizers or saddle points of the corresponding energy functional. We obtain multiple geometrically distinct solutions when this functional is invariant with respect to some discrete group. Among other invoked features in ensuring the existence of critical points, the impact of the boundary condition in relation with the singular character of the operator is emphasized. The discussion is completed by addressing the solvability of a general non-potential system with relativistic operator, subject to a non-potential boundary condition expressed in terms of a maximal monotone operator. This talk is based on the recent results in [1-3].

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FIXED POINT ITERATIVE TECHNIQUES FOR SOLVING NONLINEAR CONVEX OPTIMIZATION PROBLEMS

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Fixed point theory has long played a crucial role in analyzing a wide range of nonlinear convex optimization problems, many of which arise from practical applications. Its relevance spans several fields, including image processing, artificial intelligence, machine learning, control systems, and dynamic optimization. This presentation focuses on the development and application of fixed point iterative methods for convex optimization, specifically perturbed gradient-based and proximal-based schemes. Such methods have ensured effectiveness in solving optimization problems commonly encountered in linear inverse models, such as image restoration. Emphasis is placed on designing algorithms that are both computationally efficient and theoretically sound. The geometric structure of these methods is analyzed using standard inequalities and geometric inequalities associated with Hilbert spaces.

[1] *Speaking author*: P. Kumam

A CONVERGENCE CRITERION FOR HISTORY-DEPENDENT VARIATIONAL INEQUALITIES

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We consider a variational inequality in a reflexive Banach space X, governed by a history-dependent operator. The existence of a unique solution to the inequality is proved by using a fixed point argument. Based on the fixed point structure of the problem, we provide necessary and sufficient conditions which guarantee the uniform convergence of a sequence of functions to the solution. We exploit this result both in the study of a penalty method and the well-posedness analysis of the problem. Moreover, we present its application in the study of a mathematical model which describes the equilibrium of an elastic body in contact with a rigid-plastic foundation. The contact is frictionless and the hardening of the foundation is taken into account. We use our abstract results to obtain the continuous dependence of the solution with respect to various data and parameters. Finally, we use a finite element scheme to approximate the problem, implement it on the computer and provide numerical simulations which validate the theoretical convergence results.

PENALIZATION APPROACHES IN SHAPE AND TOPOLOGY OPTIMIZATION

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We consider first a simply supported plate with constant thickness, defined on an unknown multiply connected domain. We optimize its shape according to some given performance functionals. A second case of interest that we discuss is related to stationary Navier-Stokes equations. The nonlinearity of the state system introduces supplementary difficulties. Finally, we briefly comment a certain "counterexample" associated to a clamped plate model and extension to boundary cost functionals, etc.. Our methods are of fixed domain type, easy to implement, based on fictitious domain type approaches, that have a long history. The algorithms that we introduce are of gradient type and perform simultaneous topological and boundary variations. Numerical experiments are also included and show the efficiency of the proposed approaches. This work is in cooperation with Cornel Murea, Univ. Mulhouse, France.

SECTION 1: ALGEBRA, GEOMETRY AND TOPOLOGY

COMPLETENESS OF THE FACTOR GROUP OF A COMPLETE TOPOLOGICAL ABELIAN GROUP BY A COMPACT SUBGROUP

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The question of preserving the completeness of topological groups and the completeness of topological rings under various constructions is one of the areas of research in topological algebra (see, for example, [2]). This work is devoted to the study of the question of preserving the completeness of topological groups and topological rings when taking factor groups and is a continuation of the research that was presented in the articles [1], [2] and [3].

Theorem 1. *If M is a subgroup of an Abelian group $G(+)$ and τ is a group topology on the group G such that (G,τ) is a complete topological group and M is a compact subgroup of the topological group (G,τ), then the factor group $(\bar{G},\bar{\tau}) = (G,\tau)/M$ is a complete topological group.*

Theorem 2. *Let A and B be subgroups of an Abelian group G and let τ be a group topology on the group G such that the topological group (G,τ) is a complete topological group and A is a closed subgroup of the topological group (G,τ). If B is an open subgroup in the topological group $(A,\tau|_A)$ and $(B,\tau|_B)$ is a compact group, then the factor group $(G,\tau)/A$ is a complete group.*

Corollary *If a complete topological Abelian group (G,τ) contains a subgroup A such that the topological group $(A,\tau|_A)$ is a locally compact totally disconnected group, then the factor group $(G,\tau)/A$ is a complete topological group.*

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[1] *Speaking author*: V. I. Arnautov

ABOUT SA LOOPS OF ORDER 10

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This is a continuation of the research from [1,2]. A semisymmetric anticommutative loop is called *SA loop*. For example,

$\circ$	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	6	4	5	8	7	2	9	3
2	2	7	0	8	9	3	1	6	4	5
3	3	9	5	0	1	6	8	4	2	7
4	4	3	8	7	0	1	9	5	6	2
5	5	4	9	2	7	0	3	8	1	6
6	6	2	7	5	8	9	0	1	3	4
7	7	6	1	9	3	4	2	0	5	8
8	8	5	3	6	2	7	4	9	0	1
9	9	8	4	1	6	2	5	3	7	0

Semisymmetricity:
$x \circ (y \circ x) = y$ is equivalent to
$(x \circ y) \circ x = y$

Anticommutativity:
$x \circ y = y \circ x \Rightarrow$
$\Rightarrow (x = 0 \vee y = 0 \vee x = y).$

(1)

Theorem 1. *Every SA loop of order 10 is isomorphic to exactly one of the 22 loops. The list of these loops has been computed.*

Theorem2. *Each quasigroup isotopic to the given SA loop* $\mathcal{L} := (Z_{10}; \circ, 0)$ *coincides with exactly one of the loop* $(Z_{10}; f)$*, where*

$$f(x, y) = \gamma(\alpha^{-1}(x) \circ \beta^{-1}(y)),$$

γ *belongs to a cross-section of* $S_9'/Aut\mathcal{L}$*,* S_9' *is the set of all permutations* δ *of* Z_{10} *such that* $\delta(0) = 0$ *and* α, β *are permutations of* Z_{10}*.*

Corollary. *The number of quasigroups isotopic to the given SA loop* $\mathcal{L}$ *of order 10 is equal to*

$$\frac{(10!)^2 \cdot 9!}{|Aut\mathcal{L}|} = \frac{4\,778\,472\,583\,987\,200\,000}{|Aut\mathcal{L}|}.$$

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[1] *Speaking author*: B.Y. Buniak

THE BEHAVIOR OF GEODESIC ORBITS IN HYPERBOLIC SURFACES

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In this work we present a complete analysis of the behavior of geodesic orbits emanating from a point on an arbitrary hyperbolic surface. There exists a unique geodesic starting from a point in a given direction. Arbitrary hyperbolic surfaces, closed or open, of finite or infinite genus are considered. Yet another way to define a hyperbolic surface is via its universal cover. Geodesics of surfaces which start at a point in any direction can be drawn and finding by applying the new method. (See [1-3]).

How do geodesic orbits behave? The behavior of geodesics in hyperbolic manifolds has been widely studied by many authors and for many years. To the best of our knowledge, Hadamard was the first to demonstrate the existence of non-compact hyperbolic manifolds admitting non-trivial bounded geodesic orbits. Important qualitative characteristics of the corresponding geodesics are found in the present study, thus proving the complexity of their behavior in general.

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GENERALIZED MONOID RINGS AND APPLICATIONS

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Given an associative ring A, a multiplicative monoid G, a *D-structure*, σ is a set of maps $\sigma_{x,y} : A \longrightarrow A$ where $x, y \in G$, satisfying a suitable set of axioms. The concept was defined in [1], see also [2] and the term was coined in [3]. A related system, in a more general setting, was introduced in [5].

A D-structure for A and G defines a ring we call $A\langle G, \sigma\rangle$ which is a generalization of the monoid ring; for $a, b \in A$ and $x, y \in G$, $ax \cdot by$ is defined by the action of the various $\sigma_{x,z}$ on b. For the usual monoid ring we have $ax \cdot by = abxy$ and this is $A\langle G, \sigma\rangle$ when $\sigma_{x,x} = id$ for all x and $\sigma_{x,y}$ is the zero map when $x \neq y$.

This construction naturally generalizes classical Ore extensions [6], encompasses results previously obtained by Smits [7], and extends to other related structures, see [1] and [4]. The obtained results are illustrated by various types of polynomial rings and Weyl algebras, and applied to the study of skew-polynomial rings and other related domains.

Recently, Nystedt, Öinert, and Richter studied Ore extensions in the context of nonassociative rings, while Vladeva explored a variant of Ore extensions for semirings [8].

In this talk, we introduce the concept of a (generalized) skew polynomial ring over A. One approach to constructing such rings is via a D-structure, which, in this context, appears as a special case of the generalized monoid algebra concept.

Theorem. *Every skew polynomial ring can be defined by a D-structure.*

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ON A NEW QUASIGROUP PROLONGATION AND ITS RECURSIVE DIFFERENTIABILITY

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A prolongation of a finite quasigroup is a process of extending the quasigroup by adding one or more new elements and redefining the operation to create a new quasigroup of a larger order. The notion of prolongation was introduced by Belousov in 1967, although the construction of quasigroups prolongations was first considered by Bruck in 1944 [1–3]. Later, some other methods of quasigroups prolongations have been proposed by Osborn (1961), Yamamoto (1961), Denes and Pasztor (1963), Belyavskaya (1969), Derienko and Dudek (2008, 2013) and others.

Belousov's method of prolongations uses complete mappings that in finite case are equivalent to transversals of the corresponding latin square. A complete mapping of a quasigroup $(Q, \cdot)$ is a bijection $x \to \theta(x)$ of Q upon Q, such that the mapping θ_1, where $x \cdot \theta(x) = \theta_1(x)$, is a bijection as well. If θ is a complete mapping of a finite quasigroup $(Q, \cdot)$, then the set $\{(x, \theta(x)) | x \in Q\}$ is a transversal of the latin square, defined by $(Q, \cdot)$.

In the present work a new method of prolongation of finite quasigroups is proposed, which involves two transversals that intersect in exactly one cell. There exist 12 possibilities of prolongation of this type, 6 of which keep the recursive differentiability [4–7] of the initial latin square.

Also it is shown that there exist exactly 240 latin squares of order five with two transversals which intersect exactly in one cell, where one of the transversals is on the main diagonal of the latin square and has a fixed order of elements.

Necessary and sufficient conditions when the prolongation of a recursively differentiable quasigroup keeps this property are given, including for well known prolongation methods by Bruck and Belousov.

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ON HYPERBOLIC MANIFOLDS WITH FUNDAMENTAL GROUPS GENERATED BY TRANSLATIONS

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A method of construction of the fundamental polytope for a complete non-compact hyperbolic manifold of dimension 5 of finite volume is proposed [1]. Geometry of this manifold and its total geodesical submanifolds of different codimenions are investigated. Similar constructions of fundamental polytopes for manifolds with fundamental groups generated by translations (parabolic or hyperbolic) are discussed in different dimensions [2, 3].

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ON COMPACT ORIENTABLE AND NON-ORIENTABLE HYPERBOLIC MANIFOLDS WITH THE SAME FUNDAMENTAL POLYTOPES

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The communication will be devoted to proving the following result.

Teorem. For any natural number $N \geq 2$ there exists a convex bounded polytope which is a fundamental polytope for at least N different non-orientable compact hyperbolic 3-manifolds and, at the same time, for N different orientable hyperbolic 3-manifolds. And motion groups of these manifolds have the order no less than $2N$.

The construction of polytope. Let ω be a plane in H^3. Consider on the plane a regular polygon with $2(2p+1)$ sides and with angle $\pi/(2p+1)$ where $p = 2, 3, ...$ Through sides of this polygon we draw planes $\alpha_1, \alpha_2, ..., \alpha_{2(2p+1)}$ ortogonal to the plane ω and through midpoints of sides of the polygon we draw straight lines $l_1, l_2, ..., l_{2(2p+1)}$ also orthogonal the plane ω. It is clear that $l_i \in \alpha_i, i = 1, 2, ...2(2p+1)$, and denote $a_i = \alpha_i \cap \alpha_{i+1}, i = 1, 2, ..., 2(2p+1) - 1$, and $a_{2(2p+1)} = \alpha_{2(2p+1)} \cap \alpha_1$. On both sides of the plane ω on the straight lines a_i we intercept segments of length h. Through ends of these segments we draw planes β_i above the plane ω and γ_i below the plane ω, orthogonal to the straight lines a_i. The length h can be chosen such that planes β_i intersect straight lines l_i and l_{i+1} by angle $\pi/(2p+1)$ and analogusly planes γ_i intersect straight lines l_i and l_{i+1} by the same angle. It is not difficult to see that the planes β_i and $\beta_{(i+1)}$) intersect along straight lines b_i and the angle between these planes is equal to $2\pi/(2p+1)$, and that the straight lines b_i are orthogonal to the respective planes α_i. Analogously the planes γ_i and $\gamma_{(i+1)}$ intersect along straight lines g_i and the angle between these planes is egual to $2\pi/(2p+1)$, and the straight lines g_i are orthogonal to the planes α_i. It is easy to show that the straight lines $b_1, b_2, ..., b_{2(2p+1)}$ form a hyperbolic bundle of straight lines, analogously the straight lines $g_1, g_2, ..., g_{2(2p+1)}$ also form a hyperbolic bundle of straight lines. Let plane τ_1 be orthogonal to all straight lines $g_1, g_2, ..., g_{2(2p+1)}$ and plane τ_2 be orthogonal to all straight lines $b_1, b_2, ..., b_{2(2p+1)}$. Then in the intersection of the planes $\alpha_i, \beta_i, \gamma_i, \tau_1, \tau_2$ where $i = 1, 2, ..., 2(2p+1)$ we get a polytop K_p which is bounded. We will denote faces of polytope K_p using the same letters as containing them planes. $2(2p+1)$ faces of the polytope K_p hexagons. $4(2p+1)$ faces of K_p are pentagons. And faces τ_1 and τ_2 are regular polygons. The straight line l that passes through the centers of faces τ_1 and τ_2 is the rotation axis of polytope K_p with rotation angle $\pi/(2p+1)$.

Now we indicate identifications of faces of polytope K_p that give non-orientable and orientable manifolds. First we indicate identifications of faces which coincide for both non-orientable and orientable cases. For the identification of faces β_i and γ_i we use the following scheme. Each face β_i and γ_i is identified respectively with opposite face $\beta_{i+(2p+1)}$ or $\gamma_{i+(2p+1)}$ by translation. For faces τ_1 and τ_2 we can take $2p+2$ different motions that yield different manifolds.They are a translation and screw motions with distinct rotation angles and with the same translation vector. Here the rotation angles are $\theta = i\pi/(2p+1), i = 1, 2, ..., (2p+1)$. Now we indicate motions that differ in orientable and non-orientable cases.

In non-orientable case, we identify each face α_i with face $\alpha_{(i+1)}$ by motion consisting of rotation about axis l by angle $\pi/(2p+1)$ and subsequent reflection in face $\alpha_{(i+1)}$ where $i = 1, 3, ..., 2(2p+1)-1$. In orientable case, we identify each face α_i with face $\alpha_{(i+1)}$ by motion consisting of rotation about straight line l by angle$\pi/(2p+1)$ and subsequent rotation by angle π about straight line which is the intersection of face $\alpha_{(i+1)}$ with plane ϖ where $i = 1, 3, ..., 2(2p+1)-1$.

Thus we obtain two countable series of manifolds consisting of $2p+2$ distinct manifolds for any $p = 2, 3, ...$ both in in orientable and non-orientable cases. However a fundamental polytope of these manifolds is same for fixed $p = 2, 3, ...$

It is not difficult to see that isometry group of each obtained manifold contains no less than $2p+2$ different motions.

ON THE NUMBER OF HYPERBOLIC ORIENTABLE AND NON-ORIENTABLE MANIFOLDS SHARING A FUNDAMENTAL POLYTOPE WITH FINITE VOLUME

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The communication will be devoted to the proof of the following result.

Theorem. For any natural number N there exists a convex polytope with finite volume which is a fundamental polytope for at least N different non-orientable non compact polytope for at least N different non-orientable non compact hyperbolic manifolds and, at the same time, for N orientable non compact hyperbolic manifolds. And isometry groups of these manifolds have the order no less than $2N$.

The construction of the polytope. Let a plane ω be given in H^3. We consider on the plane a regular polygon with $2(2p+1)$ sides and with angle $\pi/(2p+1)$ where $p = 1, 2, ...$ Through sides of this polygon we draw

planes $\alpha_1, \alpha_2, ..., \alpha_{2(2p+1)}$ orthogonal to the plane ω and through the midpoints of sides of the polygon we draw straight lines $l_1, l_2, ..., l_{2(2p+1)}$ also orthogonal to the plane ω. It is clear that $l_i \in \alpha_i$ where $i = 1, 2, ..., 2(2p+1)$ and let $a_i = \alpha_i \cap \alpha_{i+1}, i = 1, 2, ..., 2(2P+1) - 1$, and $a_{2(2p+1)} = \alpha_{2(2p+1)} \cap \alpha_1$. On both sides of the plane ω on the straight lines a_i we intercept segments of length h and through their ends we draw planes β_i above the plane ω and γ_i below the plane ω orthogonal to the straight lines $a_i, i = 1, 2, ..., 2(2p+1)$. Then the segment length h can be chosen such that the planes β_i be parallel to the straight lines l_i and l_{i+1}, analogously the planes γ_i be also parallel to the straight lines l_i and l_{i+1}. As the straight lines $l_1, l_2, ..., l_{2(2p+1)}$ form a hyperbolic bundle of straight lines, then there exist planes τ_1 and τ_2 which are obstructing planes to the above bundle of straight lines. Let the plane τ_1 lie below the plane ω and the plane τ_2 lie above the plane ω. Then in he intersection of the planes $\alpha_i, \beta_i, \gamma_i, \tau_1, \tau_2$ we obtain a polytope K_p which is unbounded but has finite volume. We will denote faces of the polytope K_p using the same letters as containing them planes. $2(2p+1)$ faces of the polytope K_p are hexagons with two infinitely remote vertices and four proper vertices. Also there are $4(2p+1)$ triangular faces with one proper vertex and two infinitely remote vertices. And faces τ_1 and τ_2 are regular $2(2p+1)$-gons with all vertices being on the absolute. The straight line l that passes through the centers of faces τ_1 and τ_2 is the rotation axis of polytope K_p with rotation angle $\pi/(2p+1)$. Now we indicate identifications of faces of polytope K_p that give non-orientable and orientable manifolds. First we indicate identifications of faces which coincide for both orientable and non-orientable cases. We identify faces β_i and γ_i by the following scheme. Each face β_i and γ_i is identified with opposite face $\beta_{i+(2p+1)}$ or $\gamma_{i+(2p+1)}$ respectively by translation. For faces τ_1 and τ_2 we can take $(2p+2)$ different identifications that yield different manifolds. They are a translation and screw motions with the same translation vector but distinct rotation angles where angles are $\theta = i\pi/(2p+1), i = 1, 2, ..., 2p+1$. Faces α_i will be identified in different ways in orientable and non-orientable cases. In non-orientable case, each face α_i is identified with face α_{i+1} by motion consisting of rotation about axis l by angle $\pi/(2p+1)$ and subsequent reflection in face α_{i+1} where $i = 1, 3, ..., 2(2p+1)-1$. In orientable case each face α_i is identified with face α_{i+1} by motion consisting of rotation about straight line l by angle $\pi/(2p+1)$ and rotation about straight line which is the intersection of face α_{i+1} with plane ω by angle π where $i = 1, 3, ..., 2(2p+1) - 1$. Then taking quotient space of H^3 by all the above motions we obtain two countable series of manifolds which contain $2p+2$ distinct manifolds for $p = 1, 2, ...$ both in orientable and non-orientable cases. However a fundamental polytope of these manifold is the same for fixed p. It is not difficult to see that isometry group of each obtained manifold contains no less than $2p+2$ different motions.

LOGICAL SCHEMES OF SOME ASYMMETRIC QUASIGROUPS FOR LW-CRYPTOGRAPHY

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To implement quasigroups in hardware for lightweight cryptography, such minimized logical formulas are needed so that the hardware complexity is the smallest. The main purpose is to find logical schemes of asymmetric quasigroups and calculate their hardware complexity for use in low-resource cryptography.

Let $(Z_2^2; f)$ be an asymmetric [1] quasigroup [2], where $Z_2^2 := \{00; 01; 10; 11\}$. All parastrophes left ${}^{\ell}f$, right ${}^{r}f$ and dual to them ${}^{s}f$, ${}^{s\ell}f$, ${}^{sr}f$ are found:

f	00	01	10	11
00	01	11	10	00
01	10	00	01	11
10	00	10	11	01
11	11	01	00	10

${}^{\ell}f$	00	01	10	11
00	10	01	11	00
01	00	11	01	10
10	01	10	00	11
11	11	00	10	01

${}^{r}f$	00	01	10	11
00	11	00	10	01
01	01	10	00	11
10	00	11	01	10
11	10	01	11	00

${}^{s}f$	00	01	10	11
00	01	10	00	11
01	11	00	10	01
10	10	01	11	00
11	00	11	01	10

${}^{s\ell}f$	00	01	10	11
00	10	00	01	11
01	01	11	10	00
10	11	01	00	10
11	00	10	11	01

${}^{sr}f$	00	01	10	11
00	11	01	00	10
01	00	10	11	01
10	10	00	01	11
11	01	11	10	00

Using the Quine-McCluskey method [3], the following logical formulas ${}^{\sigma}L$ were obtained for each parastrophe of the quasigroup ${}^{\sigma}f$ as two formulas $(z_1; z_2)$, where $z_1 := x_1y_1$, $z_2 := x_2y_2$ and $\sigma := \{\iota; \ell; r; s; s\ell; sr\}$.

$$L = (x_2 \oplus (y_1 \oplus y_2); \overline{x_1 \oplus (x_2 \oplus y_1)}); \quad {}^{s}L = (x_1 \oplus (x_2 \oplus y_2); \overline{x_1 \oplus (y_1 \oplus y_2)});$$

$${}^{\ell}L = (\overline{x_1 \oplus (x_2 \oplus y_2)}; x_1 \oplus (y_1 \oplus y_2)); \quad {}^{sl}L = (\overline{x_2 \oplus (y_1 \oplus y_2)}; x_1 \oplus (x_2 \oplus y_1));$$

$${}^{r}L = (\overline{x_1 \oplus (x_2 \oplus y_2)}; \overline{x_1 \oplus (y_1 \oplus y_2)}); \quad {}^{sr}L = (\overline{x_2 \oplus (y_1 \oplus y_2)}; \overline{x_1 \oplus (x_2 \oplus y_1)}).$$

Taking into account the values of logical structural elements from [4], the Latin squares complexity for hardware is

$$L = {}^{s}L = {}^{\ell}L = {}^{s\ell}L = 10.01 \text{ GE}, \qquad {}^{r}L = {}^{sr}L = 10.68 \text{ GE},$$

where $(\oplus)$ denotes XOR and the value GE (Gate Equivalent) denotes a unit of measurement that determines the manufacturing complexity of a technology regardless of the complexity of the digital electronic circuits.

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ISOMORPHISMS OF PROJECTIVE PLANES AND CORRESPONDING TRANSFORMATIONS OF COORDINATING PAIR LOOPS

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The problem of describing isomorphisms of projective planes in terms of their coordinating algebraic structures (Hall ternary rings, strictly 2-transitive sets of permutations etc.) is well known. Hall ternary rings are considered in such terms in [1,2].

This problem is studied by the author using the notions of a pair loop and a loop transversal in a loop to a suitable subloop, introduced in articles [3,4]. For simplicity, a new concept of an extended pair loop is introduced. This allows us to formulate the following criterion.

Theorem. *Two projective planes are isomorphic if and only if the corresponding extended pair loops are isotopic (and the isotopy has some special form).*

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ON THE CHARACTERIZATION OF EXTRA POLYLOOP AND ITS APPLICATIONS TO GENETIC INTERACTIONS

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This study explores the application of non-associative algebraic hyperstructures in modeling genetic interactions, where the traditional associative algebraic frameworks may be insufficient. Non-associative algebraic hyperstructures, such as extra polyloop-I, extra polyloop-II, and extra polyloop-III, provide a flexible mathematical framework for capturing complex biological relationships, including gene regulatory networks, epistasis, and metabolic pathway interactions. We investigate how algebraic properties like flexibility, commutativity, anti-commutativity, weak associativity, and power associativity can describe non-linear dependencies in genetic systems. Additionally, we examine a novel algebraic formulation for genotype-phenotype mapping to represent mutation selection processes. The population genetics and synthetic biology illustrate the utility of non-associative structures in predicting emergent genetic behaviour. Our results suggest that non-associative algebraic hyperstructures offer a powerful tool for understanding genetic complexity, with potential applications in evolutionary biology, genetic engineering, and personalized medicine.

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ON LCA GROUPS WHOSE CLOSED POLYTHETIC SUBGROUPS HAVE COMMUTATIVE RING OF CONTINUOUS ENDOMORPHISMS

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Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, we denote by $E(X)$ the ring of all continuous endomorphisms of X and by $S(X)$ the set of those prime numbers p for which X_p, the topological p-primary component of X, is non-zero. For any prime p and positive integer n, we denote by $\mathbb{Z}(p^n)$ the cyclic group of order p^n and by $\mathbb{Z}(p^\infty)$ the quasi-cyclic group corresponding to p, both taken with the discrete topology. Further, we denote by $\mathbb{Z}_p$ the group of p-adic integers with its unique compact topology, and by $\mathbb{Q}_p$ the group of p-adic numbers with its usual locally compact topology. Finally, $\mathbb{Q}$ stands for the discrete group of rational numbers, and $\mathbb{Q}^*$ for its character group with its usual compact topology.

Definition 1. A group $X \in \mathcal{L}$ is said to be polythetic if it contains a dense finitely generated subgroup.

Theorem 1. For a topological torsion group $X \in \mathcal{L}$, the following statements are equivalent:

(i) Every closed polythetic subgroup of X has commutative ring of continuous endomorphisms.

(ii) For each $p \in S(X)$, X_p is topologically isomorphic with one of the groups $\mathbb{Q}_p$, $\mathbb{Z}_p$, $\mathbb{Z}(p^\infty)$ or $\mathbb{Z}(p^{n_p})$ for some $n_p \in \mathbb{N}$.

Theorem 2. Let $X \in \mathcal{L}$ be compact and connected. The following statements are equivalent:

(i) Every closed polythetic subgroup of X has commutative ring of continuous endomorphisms.

(ii) X is topologically isomorphic to a quotient of $\mathbb{Q}^*$ by a closed subgroup.

ON 4-QUASIGROUPS WITH EXACTLY TEN DISTINCT PARASTROPHES

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An n-groupoid (Q, A) is called an n-quasigroup if each of the elements $x_1, x_2, \ldots, x_{n+1}$ in the equality $A(x_1, x_2, \ldots, x_n) = x_{n+1}$ is uniquely determined by the remaining n. The operation ${}^{\sigma}A$, defined by the equivalence

$$A(x_1, x_2, \ldots, x_n) = x_{n+1} \Leftrightarrow {}^{\sigma}A((x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}) = x_{\sigma(n+1)},$$

where $\sigma \in S_{n+1}$, is called a parastrophe of (Q, A).

The set $H = \{\sigma \in S_{n+1} | {}^{\sigma}A = A\}$, where (Q, A) is an n-quasigroup, is a subgroup of S_{n+1}. Moreover, if $\tau \in S_{n+1}$, then ${}^{\beta}A = {}^{\tau}A$ if and only if $\beta \in H\tau$. Hence, the number of distinct parastrophes of an n-quasigroup devides $(n+1)!$. Remark that every set of representatives of $\{H\tau | \tau \in S_{n+1}\}$ is a maximum set of distinct parastrophes of (Q, A).

C.C. Lindner and D. Steadly shown in [1] that finite binary quasigroups with a prescribed number of distinct parastrofes exist of every order $q \geq 4$, suggesting that this problem can be extended to n-ary quasigroups. The mentiond problem was completely solved in the ternary case, for $1, 3, 4, 6, 12$ and 24 distinct parastrophes by M. McLeish in [2]. The spectrum of ternary quasigroups with exacly 2 or 8 distinct parastrophes, is only partially described.

We study 4-ary quasigroup operations with a given maximum number of distinct parastrophes and their spectrum.

Theorem. *Let A (Q, A) be a 4-ary quasigroup, linear over an abelian group $(Q, +)$. If the subgroup $H = \{\sigma \in S_{n+1} | {}^{\sigma}A = A\}$ is isomorphic to $S_3 x Z_2$, then (Q, A) has exactly 10 distinct parastrofephes and there exist an automorphism $\alpha \in Aut(Q, +)$, $\alpha \neq I$ and an element $c \in Q$, such that the operation*

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$A(x_1, x_2, x_3, x_4)$ has one of the following forms:

$$\begin{aligned}
&\alpha(x_1) + \alpha(x_2) + \alpha(x_3) + I(x_4) + c,\\
&I(x_1) + \alpha(x_2) + \alpha(x_3) + \alpha(x_4) + c,\\
&\alpha(x_1) + I(x_2) + \alpha(x_3) + \alpha(x_4) + c,\\
&\alpha(x_1) + \alpha(x_2) + I(x_3) + \alpha(x_4) + c,\\
&\alpha(x_1) + I(x_2) + I(x_3) + \alpha(x_4) + c,\\
&I(x_1) + \alpha(x_2) + I(x_3) + \alpha(x_4) + c,\\
&I(x_1) + I(x_2) + \alpha(x_3) + \alpha(x_4) + c,\\
&\alpha(x_1) + \alpha(x_2) + I(x_3) + I(x_4) + c,\\
&\alpha(x_1) + I(x_2) + I\alpha(x_3) + I(x_4) + c,\\
&I(x_1) + \alpha(x_2) + \alpha(x_3) + I(x_4) + c,
\end{aligned}$$

where $I(x) = -x, \forall x \in Q$.

Corollary. *There exist linear 4-ary quasigroupps with exactly ten distinct parastrophes of every order $q \geq 3$.*

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ON EXISTENTIAL EXPRESSIBILITY OF TERMS IN A TOPOLOGICAL BOOLEAN ALGEBRAS WITH 2 OPEN ELEMENTS

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Expressibility of some boolean functions via other boolean functions was studied by E. Post [1]. An alternative way in studying the expressibility of Boolean functions was proposed by A.I. Maltsev [2].

The outstanding Moldovan researcher A.V. Kuzentsov proposed in [3] to consider the existential expressibility of functions of the general k-valued logic.

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Topological boolean algebras are known to have strong connections to modal logic. We consider the 4-valued topological boolean algebra $\mathfrak{B}$ with 0 and 1 as the only open elements.

Consider first-order formulas over terms of $\mathfrak{A}$ that involve logical operators $\wedge$, $\vee$, $\rightarrow$, and a binary predicate symbol $\approx$. A term A of a universal algebra $\mathfrak{A}$ is said to be *existentially expressible via the system of terms* Σ *on* $\mathfrak{A}$ [4], if: (a) there are positive integers l, m, k, (b) variables $\pi, \pi_1, \ldots, \pi_l \in Var \setminus Var(A)$, (c) terms B_{ij}, C_{ij}, D_t $(i = 1, \ldots, m,\ j = 1, \ldots, k,\ t = 1, \ldots, l)$ such that: (i) B_{ij}, C_{ij} are expressible via Σ on $\mathfrak{A}$, (ii) $\pi, \pi_1, \ldots, \pi_l \notin Var(D_i)$, $(i = 1, \ldots, l)$, and (iii)

$$\vDash (A \approx \pi) \rightarrow (\vee_{j=1}^{k} \wedge_{i=1}^{m} (B_{ij} \approx C_{ij}))[\pi_1/D_1] \ldots [\pi_l/D_l],$$
$$\vDash (\vee_{j=1}^{k} \wedge_{i=1}^{m} (B_{ij} \approx C_{ij})) \rightarrow (A \approx \pi).$$

A term $A(p_1, \ldots, p_n)$ is said to **conserve on** $\mathfrak{A}$ **the relation** R (compare with [3]) if, for any elements $\alpha_{ij} \in \mathfrak{A}$ $(i = 1, \ldots, n;\ j = 1, \ldots, s)$, the facts $\vDash R(\alpha_{i1}, \ldots \alpha_{is})$ imply $\vDash R(F[\alpha_{11}, \ldots, \alpha_{1n}], \ldots, F[\alpha_{s1}, \ldots, \alpha_{sn}]))$. Also, the system of terms Σ is said **to conserve the relation** R **on** $\mathfrak{A}$ if any term of Σ conserves R on $\mathfrak{A}$.

A set of terms Σ on algebra $\mathfrak{A}$ is said to be maximal relative to existential expressibility if there is at least a term F in $\mathfrak{A}$ that is not existentially expressible via Σ, and any term G of $\mathfrak{A}$ is existentially expressible via $\Sigma \cup G$ on $\mathfrak{A}$.

Theorem. *There are at least 6 sets of terms of the algebra* $\mathfrak{B}$*, that are maximal relative to existential expressibility sets of terms on* $\mathfrak{B}$*, and are defined as sets of terms that conserve on* $\mathfrak{B}$ *some unary relations.*

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ON THE GENERALIZED TECHNICAL CONDITION OF PROPER OPERATION

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We denote with $\mathbf{B} = \{0, 1\}$ the binary Boole algebra. The discrete time Boolean asynchronous systems are functions $\Phi : \mathbf{B}^n \to \mathbf{B}^n$ whose coordinates $\Phi_1, ..., \Phi_n$ iterate independently on each other. Timelessly, $\lambda \in \mathbf{B}^n$ exists for which the next state of $\mu \in \mathbf{B}^n$ is given by the function $\Phi^\lambda : \mathbf{B}^n \to \mathbf{B}^n, \forall i \in \{1, ..., n\}$,

$$\Phi^\lambda(\mu)_i = \begin{cases} \Phi_i(\mu), \ if \ \lambda_i = 1, \\ \mu_i, \ if \ \lambda_i = 0, \end{cases}$$

called $\lambda-$iterate. And timefully, we denote with Π_n the set of the sequences $\alpha = \{\alpha^k | \alpha^k \in \mathbf{B}^n, k \in \mathbf{N}\}$ that fulfill

$$\forall i \in \{1, ..., n\}, \text{ the set } \{k | k \in \mathbf{N}, \alpha_i^k = 1\} \text{ is infinite.}$$

Then the state function $\phi^\alpha(\mu, \cdot) : \mathbf{N} \to \mathbf{B}^n$ of the system with the initial state μ at the time instant $k \in \mathbf{N}$ is defined as

$$\phi^\alpha(\mu, k) = \begin{cases} \mu, \ if \ k = 0, \\ \Phi^{\alpha^{k-1}}(\phi^\alpha(\mu, k-1)), \ if \ k \geq 1. \end{cases}$$

The set of the states which are reachable from μ is defined by

$$O^+(\mu) = \{\phi^\alpha(\mu, k) | k \in \mathbf{N}, \alpha \in \Pi_n\}$$

and the generalized technical condition of proper operation (tcpo) of Φ with respect to μ means the fulfillment of the next property: $\forall \nu \in O^+(\mu), \forall \lambda \in \mathbf{B}^n$,

$$\Phi^\lambda(\nu) \neq \Phi(\nu) \Longrightarrow \Phi(\Phi^\lambda(\nu)) = \Phi(\nu).$$

The aim of this presentation is to give some properties of these systems.

USING SYMMETRY ELEMENTS ON HYPERBOLIC RIEMANN SURFACES

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Symmetry elements are used in mathematical crystallography and discrete geometry. A symmetry element is an auxiliary geometric object associated with a motion (isometry) of the Euclidean plane or space which characterizes its generated cyclic group (see [1]).

In a similar way some symmetry elements can be indicated for isometries of the hyperbolic plane (and space). In [2, 3] we investigated isohedral tilings on Riemann surfaces of genus two. Such a Riemann surface can be obtained as the quotient space of the hyperbolic plane $\mathbb{H}^2$ by a translation group.

Isometries of the hyperbolic plane $\mathbb{H}^2$ induce by covering map some automorphisms of the Riemann surface. Correspondingly the images of symmetry elements on the hyperbolic plane $\mathbb{H}^2$ by the covering map serve as symmetry elements on the Riemann surface. A collection of rotation centers and an angle are associated with a rotation v. A collection of geodesics is associated with a reflection m.

The principle of symmetry elements [1] allows us to use visual geometric objects in our reasoning.

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SECTION 2: ANALYSIS, DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS

CODIMENSION IN PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS

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In this work we are concerned with the concept of codimension for planar polynomial vector fields.

This notion occurs in various articles applied to specific cases such as for example a cusp singularity of codimension 2 or a codimension 1 homoclinic loop of a saddle point with non-zero trace. We needed a rigorous, general definition that could be used efficiently not just for small codimensions as it usually occurs in the literature. For global problems such as the topological classification problem of planar quadratic vector fields modulo limit cycles we need to deal with higher codimensions and handling higher codimensions becomes quite challenging as the number of possibilities increases. We extend Sotomayor's definition based on the topological equivalence relation, allowing other equivalence relations such as for example the geometric one. We need to apply it for more general objects than singularilies or homoclinic loops, as global configurations of singular points, more general graphics and even phase portraits.

Finally we apply our extended definition of codimension to assign a specific codimension to all 207 global topological configurations of singularities of quadratic differential systems excepting those with centers.

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PREDICTING PREDATOR EXTINCTION IN SPACE AND TIME

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This study investigates the eradication of an invasive predator population within an environment subject to seasonal variation (see [1]). This is a problem of growing significance in ecological management and conservation biology. We consider a general prey–predator model that includes nonlocal reaction terms, local diffusion, and time-periodic coefficients (see [2, 3]), enabling us to capture the intricate dynamics of such ecosystems.

Our analysis focuses on the role of selective control strategies in achieving long-term eradication. These strategies target the predator population either *directly*, through an impulsive or harvest-type control, or *indirectly*, by applying a harvest-type control to the prey population.

These results offer new insights into species management in temporally and spatially dynamic systems and lay the groundwork for the development of more targeted, ecologically grounded intervention strategies.

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MULTIFREQUENCY SYSTEMS WITH AN ASYMPTOTIC HIERARCHY OF AMPLITUDE AND PHASE VARIABLES

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A system of oscillators with nonlinear couplings characterized by different asymptotics of a small parameter is studied. The system of equations takes the following form:

$$\frac{d^2u_\nu}{dt^2} + w_\nu^2(\tau_\nu)u_\nu = \varepsilon^{\kappa_\nu} f_\nu(\tau_\nu, u, \frac{du}{dt}),\ i = \overline{1,n}\ , \tag{1}$$

where $\tau_\nu = \varepsilon^{\kappa_\nu} t$, $0 < \kappa_1 < ... < \kappa_n$, $u = (u_1, ..., u_n)^T$, ω_ν - natural frequencies of the oscillators when $\varepsilon = 0$.
By applying the change of variables:

$$u_\nu = a_\nu cos\varphi_\nu,\ \frac{du_\nu}{dt} = -a_\nu\omega_\nu sin\varphi_\nu$$

system (1) is reduced to the form:

$$\begin{aligned} \frac{da_\nu}{d\tau_\nu} &= X_\nu(\tau_\nu, a, \varphi), \\ \frac{d\varphi_\nu}{d\tau_\nu} &= \frac{\omega_\nu(\tau_\nu)}{\varepsilon^{\kappa_\nu}} + Y_\nu(\tau_\nu, a, \varphi), \end{aligned} \tag{2}$$

where $\nu = \overline{1,n}$, $a = (a_1, ..., a_n)^T$, $\varphi = (\varphi_1, ..., \varphi_n)^T$.

Similar systems of equations were considered in [1-2]. The system of equations averaged over the phase variables [3] takes the form:

$$\begin{aligned} \frac{d\overline{a}_\nu}{d\tau_\nu} &= X_{\nu,0}(\tau_\nu, \overline{a}), \\ \frac{d\overline{\varphi}_\nu}{d\tau_\nu} &= \frac{\omega_\nu(\tau)}{\varepsilon^{\kappa_\nu}} + Y_{\nu,0}(\tau_\nu, \overline{a}). \end{aligned} \tag{3}$$

The system (3) is significantly simpler than system (2), as the equations for the amplitude variables $\overline{a}_\nu$ do not depend on the phase variables $\overline{\varphi}_\nu$. The averaging method is justified, and estimates for the deviations of the amplitude

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and phase variables are obtained, explicitly depending on the small parameter ε and values κ_ν. The results are illustrated by an example.

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A FAMILY OF CUBIC SYSTEMS WITH INVARIANT STRAIGHT LINES IN THE CONFIGURATION OF THE TYPE $(3, 2, 1)$

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We focus on the class $\mathbf{CSL_7}$ of non-degenerate real planar cubic vector fields with invariant straight lines of total multiplicity 7, including the line at infinity with its own multiplicity. In addition they possess three real distinct infinite singularities, defined by the linear factors of the polynomial $C_3(x, y) = yp_3(x, y) - xq_3(x, y)$, where p_3 and q_3 are the cubic homogeneities of these systems.

In [1] the notions of *configuration of invariant lines* and *type of configuration* were defined. Here we consider the subfamily $\mathbf{CSL}^{\mathbf{3r}\infty}_{\mathbf{(3,2,1)}}$ of cubic systems possessing only the configurations of the type $(3, 2, 1)$ which possess three real distinct infinite singularities. We say that these lines form a configuration of the type $(3, 2, 1)$ if there exist one triplet, one couple of parallel invariant straight lines (real or complex, which may coincide) and one single invariant line, every set with different slopes, taking into consideration singular points situated on these lines and their multiplicities.

We proved that if the infinite singularities of a cubic system are determined by one double and two simple factors of C_3 and possesses the configuration $(3, 2, 1)$ then this system must have the homogeneities (p_3, q_3): $(x^3 - x^2y - xy^2, -y^3)$ or $(-x^2y, -x^2y)$ or (x^3, xy^2). We determine that systems belonging

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to this family with the cubic homogeneities (x^3, xy^2) could not have invariant straight lines in the configuration of the type $(3, 2, 1)$.

Our main result is the following one: *a cubic system belonging to the family* $\mathbf{CSL}^{\mathbf{3r}\infty}_{\mathbf{(3,2,1)}}$ *could have only one of 10 distinct configurations of invariant straight lines. For each one of these configurations we present corresponding example for its realization.*

We remark that the maximum multiplicity of an invariant straight line (finite or infinite) is 2 and the maximum multiplicity of a real finite singular point is 4.

It is worth mentioning that cubic systems with invariant straight lines along three directions of total multiplicity 8 was studied in [2]. Cubic systems with the maximum number of invariant straight lines, i.e. 9, was studied in [3].

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ON THE ALGEBRAIC CLOSENESS OF THE SPACE OF REMOTELY ALMOST PERIODIC FUNCTIONS

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Let $\mathbb{R}$ (respectively, $\mathbb{C}$) be the set of all real (respectively, complex) numbers. Denote by $C(\mathbb{R}, \mathbb{C})$ the family of all continuous functions $\varphi : \mathbb{R} \to \mathbb{C}$ equipped with the compact-open topology and $(C(\mathbb{R}, \mathbb{C}), \mathbb{R}, \sigma)$ the shift dynamical system [1] on the space $C(\mathbb{R}, \mathbb{C})$.

A subset A of $\mathbb{R}$ is called relatively dense on $\mathbb{R}$ if there exists a positive number ℓ such that each segment of length ℓ contains at least one element of the set A.

Definition. A function $\varphi \in C(\mathbb{R}, \mathbb{C})$ is said to be:

1. remotely almost periodic [1] if for any $\varepsilon > 0$ there exists a relatively dense subset $\mathcal{P}(\varepsilon)$ with the property that for any $\tau \in \mathcal{P}(\varepsilon)$ we have a number $L(\varepsilon, \tau)$ such that $|\varphi(t + \tau) - \varphi(t)| < \varepsilon$ for any $t \geq L(\varepsilon, \tau)$;

2. positively Lagrange stable [1] if the set $\sum_{\varphi}^{+} := \{\varphi^h |\ h \geq 0\}$ is precompact in the space $C(\mathbb{R}, \mathbb{C})$.

Let

$$x^n + a_1(t)x^{n-1} + \ldots + a_{n-1}(t)x + a_n(t) = 0 \tag{1}$$

be an algebraic equation of degree n with coefficients $a_1, \ldots, a_n \in C(\mathbb{R}, \mathfrak{C})$.

Theorem 1. Suppose that the following conditions hold:

1. $\lambda \in C(\mathbb{R}, \mathbb{C})$ is a solution of the equation (1);
2. the function $a := (a_1, \ldots, a_n) \in C(\mathbb{R}, \mathbb{C}^n)$ is remotely almost periodic and positively Lagrange stable;
3. the omega limit set $\omega_{(a_1,\ldots,a_n)}$ of $(a_1, \ldots, a_n)$ is minimal;
4. the discriminant $D(t) = D[a_1(t), \ldots, a_n(t)]$ of the equation (1) is separated from zero on the semi-axis $\mathbb{R}_+ := \{t \in \mathbb{R} |\ t \geq 0\}$, i.e., $\inf\limits_{t \geq 0} |D(t)|\} > 0$.

Then the continuous solutions $\lambda = \lambda(t)$ of the equation(1) is remotely almost periodic.

Open problem. The question is whether Theorem 1 remains true in the general case when the set $\omega_{(a_1,\ldots,a_n)}$ is not minimal, it remains open.

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CENTER CONDITIONS FOR A CUBIC SYSTEM WITH ONE INVARIANT STRAIGHT LINE AND ONE INVARIANT CONIC

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We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x,y) + p_3(x,y),\ \dot{y} = -x + q_2(x,y) + q_3(x,y), (1)$$

where $p_j(x,y)$, $q_j(x,y) \in \mathbb{R}[x,y]$, $j \in \{2,3\}$ are homogeneous polynomials of degree j. The origin $O(0,0)$ is a singular point for system (1) with purely imaginary eigenvalues ($\lambda_{1,2} = \pm i$), i.e. it is a focus or a center. The problem of distinguishing between a center and a focus (the problem of the center) is open for general cubic systems.

An approach to the problem of the center is to study the local integrability of the system (1) in some neighborhood of $O(0,0)$. The singular point $O(0,0)$ is a center for cubic system (1) if and only if the system has an analytic integrating factor of the form $\mu(x,y) = 1 + \sum_{k=1}^{\infty} \mu_k(x,y)$ in a neighborhood of $O(0,0)$, where μ_k are homogeneous polynomials of degree k.

Integrability conditions provided by the existence of Darboux integrating factors where obtained for some families of cubic systems (1) with algebraic solutions: four and three invariant straight lines [1]; two parallel invariant straight lines [2]; two invariant straight lines and one invariant conic [1]; one invariant straight line and one invariant cubic [3]; one invariant cubic [4].

The problems we consider in this talk are the following:

1) Find the subclass of cubic differential systems (1) which has one invariant straight line $l \equiv a_1x + b_1y + 1 = 0, (a_1, b_1) \neq 0$ and one invariant irreducible conic $\Phi \equiv a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{10}x + a_{01}y + 1 = 0$, $(a_{20}, a_{11}, a_{02}) \neq 0$.

2) For this subclass, determine the integrability conditions by constructing Darboux integrating factors of the form $\mu = l^{\alpha}\Phi^{\beta}$, $\alpha\beta \neq 0$.

For the cubic system (1) with algebraic solutions $l = 0$ and $\Phi = 0$, twelve sets of conditions for a singular point $O(0,0)$ to be a center were obtained [5].

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MULTIDIMENSION RIEMANN METRICS TO INTEGRATING OF THE NAVIER-STOKES EQUATIONS

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Theorem 1. *The metric of the form*

$$ds^2 = 2\,dx^2 + 2\,dxdy + 2\,dxdu + 2\,dy^2 + 2\,dydz + 2\,dydv + 2\,dz^2 +$$
$$+2\,dzdw + 2\,dtdp + 2\,d\eta d\xi + 2\,d\rho d\chi + 2\,dmdn +$$
$$+Adt^2 + Bd\eta^2 + Cd\rho^2 + Edm^2, \tag{1}$$

where

$$A = 2 - U(x,y,z,t)\,u - V(x,y,z,t)\,v - W(x,y,z,t)\,w,$$
$$B = \left(-UW + \mu\frac{\partial}{\partial z}U\right)w + \left(-UV + \mu\frac{\partial}{\partial y}U\right)v + \left(\mu\frac{\partial}{\partial x}U - (U)^2 - P\right)u - Up,$$
$$C = \left(-VW + \mu\frac{\partial}{\partial z}V\right)w + \left(\mu\frac{\partial}{\partial y}V - (V)^2 - P\right)v + \left(-UV + \mu\frac{\partial}{\partial x}V\right)u - Vp,$$
$$E = \left(-\mu\frac{\partial}{\partial x}U - \mu\frac{\partial}{\partial y}V - (W)^2 - P\right)w + \left(-VW + \mu\frac{\partial}{\partial y}W\right)v +$$
$$+\left(-UW + \mu\frac{\partial}{\partial x}W\right)u - Wp$$

is the Ricci-flat metric on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{U} + \left(\vec{U}\cdot\vec{\nabla}\right)\vec{U} - \mu\Delta\vec{U} + \vec{\nabla}P = 0, \qquad \vec{\nabla}\cdot\vec{U} = 0, \tag{2}$$

where $\vec{U} = [U(\vec{x},t), V(\vec{x},t), W(\vec{x},t)]$ *is the fluid velocity,* $P = P(\vec{x},t)$ *is the pressure,* μ *is the viscosity and* $\vec{x} = (x,y,z)$.

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Proposition. *The metrics (1) and*

$$y^2 {d_x}^2 + 2\left(l\,(x,z)\,y^2 + m(x,z)\right)d_x d_z + 2\,d_y d_z + \left(\left(l(x,z)\right)^2 y^2 - 2\left(\frac{\partial}{\partial x} l(x,z)\right)y + 2\,l\,(x,z)\,m(x,z) + 2\,l\,(x,z)\right){d_z}^2 \tag{3}$$

that are applied to integration of the KdV-equation:

$$\frac{\partial}{\partial z} l\,(x,z) - 3\,l\,(x,z)\,\frac{\partial}{\partial x} l\,(x,z) + \frac{\partial^3}{\partial x^3} l\,(x,z) = 0,$$

the KP-equation:

$$\frac{\partial^2}{\partial x \partial t} n\,(x,y,t) - 3\left(\frac{\partial}{\partial x} n\,(x,y,t)\right)^2 - n\,(x,y,t)\,\frac{\partial^2}{\partial x^2} n\,(x,y,t) + \frac{\partial^4}{\partial x^4} n\,(x,y,t) - \alpha^2 \frac{\partial^2}{\partial y^2} n\,(x,y,t) = 0,$$

the Schrödinger equation:

$$i\frac{\partial}{\partial t}\phi\,(x,y,t) - \kappa\,\frac{\partial^2}{\partial x^2}\phi\,(x,y,t) - K(x,y,t)\phi\,(x,y,t) = 0,$$

may be used to construct an example of exact solutions of the system (2).

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SHADOWING IN LINEAR SKEW-PRODUCT FLOWS

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Shadowing theory begins with Anosov's "lemma on epsilon-trajectories" (1967), which is concerned with geodesic flows on negatively curved closed Riemannian manifolds, and, independently, with Bowen's (1975) Axiom A diffeomorphisms. In both situations this phenomenon is related to one of "hyperbolic" features of dynamical systems, which is invariant under the topological conjugacy. It generally refers to the crucial property of hyperbolic sets, which means that any pseudo-orbit with possible small "errors" in the iteration of the map can be approximated by a genuine orbit.

Since then various types of shadowing properties have been defined for different purposes: linear operators, random dynamical systems, set-valued dynamics and semigroup actions, including Iterated Function Systems, etc.

We are concerned with "partial shadowing" in linear, or affine, non autonomous dynamical systems in discrete time. The phase space of such a system consists of a real vector bundle with compact base. The dynamics is generated by the iterations of an (linear on the fibers) isomorphism of the vector bundle covering the iterations of a homeomorphism in the base. Any pseudo orbit with possible "errors" (jumps in the same fiber) is approximated by an appropriately chosen genuine orbit, both projecting on the same orbit in the base.

We give precise definitions of various types of shadowing, and establish criteria for a linear skew-product flow to have the shadowing property in terms of weak forms of exponential dichotomy as, e.g., transversality, or (weak) regularity.

We also purpose some applications of these results to Iterated Function Systems, random dynamical systems, and linear operators.

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HALF A CENTURY OF FIXED POINT THEORY RESEARCH IN THAILAND: A BIBLIOMETRIC ANALYSIS

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Fixed point theory is a fundamental area of mathematical research with diverse applications in analysis, optimization, and computational mathematics. Over the past five decades, Thailand has made notable contributions to this field, reflected in a growing number of publications, collaborations, and scholarly impact. This study presents a comprehensive bibliometric analysis of fixed-point theory research in Thailand. Using data from the Scopus database, analysed with VOS viewer software, we examine publication trends, citation patterns, leading authors, key institutions, and international collaborations. Our findings reveal significant growth, with total publications increasing from 35 (1973–2005) to 2,332 (2000–2024). Among the leading contributors, Poom Kumam from King Mongkut's University of Technology Thonburi (KMUTT) stands out with 460 publications and 7,094 citations, while KMUTT itself is the most prolific institution, producing 512 papers. Collaboration analysis identifies Saudi Arabia as the primary international partner, contributing 235 co-authored works, and Sotiris K. Ntouyas from the University of Ioannina, Greece, as a key collaborator. Early studies were centred on foundational concepts, recent research has increasingly explored applied domains. Topical and keyword analyses indicate that research in Thailand is somewhat less aligned with global trends, especially towards artificial intelligence and machine learning. These findings provide valuable insights into the evolution and impact of fixed-point theory research in Thailand, offering guidance for future research directions and policy development in the mathematical sciences.

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QUANTIZATION AND LIOUVILLE TYPE PROPERTIES FOR THE SYSTEM $-\Delta u = uJ'(1-|u|^2)$ WITH A POTENTIAL CONVEX NEAR ZERO

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This is a joint work with Umberto De Maio, Rejeb Hadiji and Carmen Perugia.

We consider in R^2 a Ginzburg-Landau type equation

$$-\Delta u = uJ'(1-|u|^2),$$

and we extend the results concerning quantization of finite potential solutions, obtained in [1], to general potential functions J satisfying weak conditions allowing for example a zero of infinite order in the origin.

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FUNCTIONAL BASES OF CENTER-AFFINE INVARIANT POLYNOMIALS FOR THE DIFFERENTIAL SYSTEM $s^3(1,2)$

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We will examine the differential system $s^3(1,2)$ of the form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta} x^\alpha x^\beta \quad (j,\alpha,\beta = \overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta}$ is a symmetric tensor in lower indices in which the total convolution is done. Note that the system (1) contains, as particular cases, the differential systems $s^3(1)$ and $s^3(2)$ with homogeneities of degree 1 and 2, respectively.

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Based on the ideas from the monographs [1–6], these systems were examined in the works [7, 8] under different aspects. However, for the mentioned systems, the problem of constructing the functional bases of invariants, comitants, contravariants and mixed comitants (see, for example [1–7]), is not completely solved.

By means of the Lie algebras of the differential operators, admitted by the systems $s^3(1)$, $s^3(2)$ and $s^3(1,2)$ constructed in [7], it was proved

Theorem 1. *The number of the invariant center-affine polynomials of the systems $s^3(1)$, $s^3(2)$, $s^3(1,2)$ that form the functional bases of invariants $\mu(I)$, covariants $\mu(K)$, contravariant $\mu(R)$ and of mixed comitants $\mu(S)$, respectively, is given in the following table:*

	$\mu(I)$	$\mu(K)$	$\mu(R)$	$\mu(S)$
$s^3(1)$	3	4	4	7
$s^3(2)$	10	13	13	16
$s^3(1,2)$	19	22	22	25

The elements of these bases were constructed.

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SOLUTION OF SOME SINGULAR INTEGRAL EQUATIONS WITH UNBOUNDED COEFFICIENTS

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Integral equations are found in various fields of science and in numerous applications (such as physics, control theory, economics and medicine). The theory of such equations was founded by A. Poincaré and D. Hilbert almost immediately after the appearance of the classical theory of Fredholm integral equations. Exact solutions of integral equations play a major role in forming a correct understanding of the qualitative characteristics of many phenomena and processes in various fields of natural science. However, as mentioned in the monographs of N. Muskelishvili and F. Gakhov and in other works, the solution of singular integral equations can be determined in rare cases, and even in these cases, the determination of an exact solution requires the calculation of singular integrals, which is accompanied by great difficulties of both theoretical and computational nature. The most often applied method for solving singular integral equations on closed contours consists in the equivalent reduction of the given equation to the Riemann-type boundary value problem, which is solved effectively. In the case of an open integration contour (a bounded or unbounded segment), this method can no longer be applied, and therefore various other methods are used depending on the given equation.

In this talk, the methods of analytical continuation of the unknown function and the coefficients of the equation to the entire complex plane, some integral transformations, as well as the theory of residues and limit transition are applied. As a result of these actions, equations of the form

$$\frac{1}{\pi i}\int_a^b \frac{\varphi(t)}{t-x}dt - \lambda^2 \frac{1}{\pi i}\int_a^b \frac{K_m(t,x)}{t-x}\varphi(t)dt = f(x) \quad (m=1,2),$$

where λ is a complex parameter, f is a given Hölder function on the interval $[a,b]$,

$$K_1(t,x) = \sqrt{\frac{(b-x)(x-a)}{(b-t)(t-a)}},\ K_2(t,x) = \sqrt{\frac{(b-x)(t-a)}{(b-t)(x-a)}},$$

are reduced to solving ordinary characteristic singular equations with the Cauchy kernel. It should be noted that when applying these methods, the Poincaré-Bertrand permutation formula in the singular integral plays a significant role:

$$\frac{1}{\pi i}\int_L \frac{d\tau}{\tau - t}\frac{1}{\pi i}\int_L \frac{\varphi(\tau,s)}{s-\tau}ds = \varphi(t,t) + \frac{1}{\pi i}\int_L ds \frac{1}{\pi i}\int_L \frac{\varphi(\tau,s)}{(s-\tau)(\tau-t)}d\tau.$$

The obtained results are compared with some known results for some equations that represent particular cases of the equations considered in this talk. As a result of these comparisons, we are convinced that the obtained results are correct.

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THE HYPERBOLIC RELAXATION OF THE NON VISCOUS CAHN-HILLIARD EQUATION

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Let $\Omega \subset \mathbb{R}^n\,(n=1,2)$ be a bounded domain with smooth boundary $\partial\Omega$, $T>0$ and $Q_T=\Omega\times(0,T)$. We will study the behaviour of the solutions of the initial boundary problem

$$\left\{\begin{array}{l} \varepsilon\,\partial_{tt}^2 u_\varepsilon+\partial_t u_\varepsilon-\Delta\big(-\Delta u_\varepsilon+a\,u_\varepsilon+B(u_\varepsilon)\big)=f_\varepsilon(x,t), \quad (x,t)\in Q_T,\\ u_\varepsilon\big|_{t=0}=u_{0\varepsilon}(x), \quad \partial_t\,u_\varepsilon\big|_{t-0}=u_{1\varepsilon}(x), \quad x\in\Omega,\\ u_\varepsilon\big|_{x\in\partial\Omega}=\Delta\,u_\varepsilon\big|_{x\in\partial\Omega}=0, \quad t\in(0,T), \end{array}\right. \tag{P_ε}$$

where $\Delta: H^2(\Omega)\bigcap H_0^1(\Omega)\mapsto L^2(\Omega)$, $a\in\mathbb{R}$ and $B(u)=|u|^p\,u$. The problem (P_ε) is the hyperbolic relaxation for the corresponding initial boundary problem for the non viscous Cahn-Hilliard equation

$$\left\{\begin{array}{l} \partial_t v-\Delta\big(-\Delta v+a\,v+B(v)\big)=f(x,t), \quad (x,t)\in Q_T,\\ v\big|_{t=0}=v_0(x), \quad x\in\Omega,\\ v\big|_{x\in\partial\Omega}=\Delta\,u\big|_{x\in\partial\Omega}=0, \quad t\in(0,T). \end{array}\right. \tag{P_0}.$$

The model (P_0) is governed by the celebrated Cahn-Hillard equation, which was proposed to describe phase separation phenomena in binary systems (phase separation process between healthy and tumour, the phase separation of a binary mixture and other phenomena) [J.W. Cahn and J.E. Hilliand, 1958]. The unknown v denotes the relative concentration of one phase or component in a binary material and B is the derivative of a non-convex potential F whose minima represent energetically more favorable configurations, usually achieved near pure phases or concentrations.

The mathematical literature devoted to (P_0) is huge and the main properties of the solutions in terms of regularity, qualitative behavior, and asymptotics are

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now well-understood. In recent years, the investigations has moved in other versions of (P_0) related to specific physical situations. Among these, is also so-called hyperbolic relaxation of the equation, proposed by P. Galenko and V. Lebedev in 2008 and which is described by the model (P_ε).

Under some smoothness conditions on the functions $u_{0\varepsilon}$, $u_{1\varepsilon}$, f_ε, v_0, f, if

$$||u_{0\,\varepsilon} - u_0||_{H_)^1(\Omega)} \to 0,\ ||u_{1\,\varepsilon} - u_1||_{L^2(\Omega)} \to 0,\ ||f_\varepsilon - f||_{W^{1,2}(0,T;L^2(\Omega))} \to 0,$$

using the relationship between solutions of the systems (P_ε) and (P_0) and a priori estimates of solutions to the system (P_ε) we prove that

$$u_\varepsilon \to v \quad \text{in} \quad C([0,T];L^2(\Omega)) \cap L^\infty(0,T;H_0^1(\Omega)), \quad \text{as} \quad \varepsilon \to 0.$$

This means that the perturbations of the system (P_0) by the system (P_ε) are regular in the indicated norms.

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THE $GL(3,\mathbb{R})$-LIE ALGEBRA ADMITTED BY THE DIFFERENTIAL SYSTEM $s^3(1,3)$

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Consider the ternary differential system in the tensorial form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma} x^\alpha x^\beta x^\gamma \ (j,\alpha,\beta,\gamma = \overline{1,3}), \tag{1}$$

where the coefficient tensor $a^j_{\alpha\beta\gamma}$ is symmetrical in lower indices in which the complete convolution holds. Let $GL(3,\mathbb{R})$ be the group of centro-affine transformations of the phase variables of the system (1).

In the School of Differential Equations from Chişinău the ternary polynomial systems of differential equations are researched using centro-affine groups.

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Theorem 1. *To the representation of the centro-affine group of transformations* $GL(3,\mathbb{R})$ *in the space of variables and coefficients of the system* (1) *correspond the Lie operators*

$$v_1 = x^1\frac{\partial}{\partial x^1} + d_1, v_2 = x^2\frac{\partial}{\partial x^2} + d_2, v_3 = x^3\frac{\partial}{\partial x^3} + d_3, v_4 = x^2\frac{\partial}{\partial x^1} + d_4, v_5 = x^3\frac{\partial}{\partial x^1} + \\ +d_5, v_6 = x^1\frac{\partial}{\partial x^2} + d_6, v_7 = x^3\frac{\partial}{\partial x^2} + d_7, v_8 = x^1\frac{\partial}{\partial x^3} + d_8, v_9 = x^2\frac{\partial}{\partial x^3} + d_9, \tag{2}$$

were d_i *are determined explicitly and represents the operators of the representation of the group* $GL(3,\mathbb{R})$ *in the space of coefficients of the system* (1).

Lemma 1. *The operators* (2) *form a nine-dimensional Lie algebra* $L_9 < v_1, ..., v_9 >$ *with structural equations*

$$[v_1, v_4] = -v_4, [v_1, v_5] = -v_5, [v_1, v_6] = v_6, [v_1, v_8] = -v_8, [v_2, v_4] = v_4, [v_2, v_6] = -v_6, \\ [v_2, v_7] = -v_7, [v_2, v_9] = v_9, [v_3, v_5] = v_5, [v_3, v_7] = v_7, [v_3, v_8] = -v_8, [v_3, v_9] = -v_9, \\ [v_4, v_6] = v_2 - v_1, [v_4, v_7] = -v_5, [v_4, v_8] = v_9, [v_5, v_6] = v_7, [v_5, v_8] = v_3 - v_1, \\ [v_5, v_9] = -v_4, [v_6, v_9] = v_8, [v_7, v_8] = -v_6, [v_7, v_9] = v_3 - v_2, \tag{3}$$

(*the rest* $[v_i, v_j] = 0$).

This algebra is the same Lie algebra as in [1] for differential system $s^3(1,2)$.

Theorem 2. *The algebra* $L_9 < v_1, ..., v_9 >$ *is a reductive Lie algebra and corresponds to the centro-affine group* $GL(3,\mathbb{R})$.

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INVARIANT PARABOLAS AND LINES CONFIGURATIONS IN PLANAR QUADRATIC SYSTEMS

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Quadratic differential systems are fundamental objects in the qualitative theory of dynamical systems. In this talk, we explore a rich class of planar quadratic systems characterized by the presence of three distinct infinite singularities and at least one invariant parabola. Specifically, we focus on the subfamily of non-degenerate systems where a key invariant, denoted by η, is nonzero—a condition ensuring this infinite singularity structure.

We present a complete classification of all possible geometric configurations involving invariant parabolas and straight lines within this family. This classification is developed modulo real affine transformations and time rescaling and is formulated in terms of affine invariant polynomials. Remarkably, the classification leads to exactly 144 distinct configurations, offering an algorithmic tool for identifying the configuration of any system in this class.

Beyond its intrinsic geometric interest, this work provides valuable insights into the algebraic and integrability properties of quadratic differential systems. The results have implications for understanding the structure of phase portraits and the broader behavior of polynomial dynamical systems.

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CENTER CONDITIONS FOR CUBIC DIFFERENTIAL SYSTEMS WITH TWO AFFINE NON-PARALLEL INVARIANT STRAIGHT LINES OF TOTAL MULTIPLICITY THREE

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We consider the real cubic differential systems of the form

$$\begin{cases} \dot{x} = y+ax^2+cxy+fy^2+kx^3+mx^2y+pxy^2+ry^3 \equiv P(x,y), \\ \dot{y} = -(x+gx^2+dxy+by^2+sx^3+qx^2y+nxy^2+ly^3) \equiv Q(x,y), \\ \gcd(P,Q)=1. \end{cases} \tag{1}$$

Let $\mathcal{L}_1$, $\mathcal{L}_2$ be invariant straight lines for (1) and $m(\mathcal{L}_1) \geq 2$, where $m(\mathcal{L}_1)$ is the multiplicity of $\mathcal{L}_1$. Then the coefficients of (1) satisfy one of the following four sets of conditions

$$\begin{array}{l} a=-(\alpha+\beta)(2+c+f\alpha+f\beta), b=-(\alpha+f\alpha\beta+f\beta^2)/\beta, \\ r=0, g=(2f\alpha+(c+2)(1+d\beta)+(\alpha+\beta)(df\beta-(c+2)\cdot \\ (\alpha-2\beta-c\beta))-f(\alpha+\beta)^2(\alpha-4\beta-2c\beta)+f^2\beta(\alpha+\beta)^3)/ \\ (f\beta), k=-a, l=f, n=-(1-\alpha^2+d\beta+c\alpha\beta)/\beta^2, p=-f, \\ q=-(df\beta^2+2\beta(d+\alpha+2\beta)+(c+2)(1-\alpha^2)+c\beta(d+3\alpha+ \\ 4\beta)+\beta(\alpha+\beta)(c^2+4f\beta+2cf\beta)+f^2\beta^2(\alpha+\beta)^2)/(f\beta^2), \\ s=\alpha(f\alpha+2d\beta+2\beta(\alpha+2\beta)+(c+2)(1-\alpha^2)+c\beta(d+3\alpha+ \\ 4\beta)-(\alpha+\beta)(f(\alpha-2\beta)(\alpha+\beta)-\beta(c^2+df))+ \\ cf\beta(\alpha+\beta)^2)/(f\beta^2), m=-c-1; \end{array} \tag{2}$$

$$\begin{array}{l} a=-(2+c)(\alpha+\beta), d=((\alpha+\beta)(\alpha-2\beta-c\beta)-1)/\beta, f=0, \\ b=-\alpha/\beta, g=(\alpha+\alpha\beta(c+1)(\alpha+\beta)-q\beta^2)/\beta, k=-a, l=0, \\ m=-c-1, r=0, n=(\alpha+2\beta+c\beta)/\beta, p=-f, \\ s=-(\alpha(q\beta+(\alpha+\beta)(\alpha+2\beta+c\beta)))/\beta; \end{array} \tag{3}$$

$$\begin{array}{l} a=c+2=f=k=m-1=p=r=0, l=-(\alpha+b\beta)/\beta^2, \\ g=(2\alpha-\alpha(\alpha+\beta)^2+\beta(d\alpha-q\beta))/\beta, \\ n=(2\alpha^2-d\beta+2\alpha\beta+b\alpha\beta-1)/\beta^2, \\ s=\alpha(\alpha-\alpha(\alpha+\beta)^2+\beta(d\alpha-q\beta))/\beta^2; \end{array} \tag{4}$$

$$\begin{array}{l} b=-(\alpha+f\alpha\beta+f\beta^2)/\beta, d=((\alpha+\beta)(\alpha-2\beta-c\beta)-1)/\beta, \\ m=-c-1, g=(\alpha-\beta-a\alpha\beta-a\beta^2)/\beta, k=-a, l=f, s=-\alpha/\beta, \\ n=(\alpha+2\beta+c\beta)/\beta, p=-f, q=(1-\alpha^2+a\beta-\alpha\beta)/\beta, r=0. \end{array} \tag{5}$$

Under the conditions (2) (respectively, (3),(4),(5)) for solving the center problem, it is necessary and sufficient that the first three (respectively, three, four [1], five) Lyapunov quantities to vanish.

Theorem. *Let the cubic system have two distinct affine non-parallel invariant straight line $\mathcal{L}_1$, $\mathcal{L}_2$ and a critical point $M_0(x_0, y_0)$ with pure imaginary eigenvalues. If $m(\mathcal{L}_1) \geq 2$, then M_0 is a center if and only if the first five Lyapunov quantity vanish* ($L_1 = L_2 = L_3 = L_4 = L_5 = 0$).

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ALMOST PERIODIC SOLUTIONS OF LATTICE DYNAMICAL SYSTEMS WITH MONOTONE NONLINEARITY

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Let $\mathbb{R}$ (respectively, $\mathbb{Z}$) be the set of all real (respectively, integer) numbers and l_2 be the Hilbert space of all sequences $\xi = (\xi_i)_{i\in\mathbb{Z}}$ ($\xi_i \in \mathbb{R}$ for any $i \in \mathbb{Z}$) with the property $\sum\limits_{i\in\mathbb{Z}} |\xi_i|^2 < \infty$ equipped with the scalar product $\langle \xi, \eta \rangle := \sum\limits_{i\in\mathbb{Z}} \xi_i \eta_i$. Denote by $C(\mathbb{R}, l_2)$ the family of all continuous functions $\varphi : \mathbb{R} \to l_2$ equipped with the compact-open topology and $(C(\mathbb{R}, l_2), \mathbb{R}, \sigma)$ the shift dynamical system on the space $C(\mathbb{R}, l_2)$.

A subset $A \subset \mathbb{R}$ is called relatively dense if there exists a positive number l such that $[a, a + l] \bigcap \mathbb{R} \neq \emptyset$, $\forall a \in \mathbb{R}$.

Definition. A function $\varphi \in C(\mathbb{R}, l_2)$ is said to be almost periodic [1] if for any $\varepsilon > 0$ there exists a relatively dense subset $\mathcal{P}(\varepsilon)$ such that $\|\varphi(t + \tau) - \varphi(t)\|_{l_2} < \varepsilon$

In this talk we study the almost periodic solutions of the systems

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ (i \in \mathbb{Z}), \tag{1}$$

where $\lambda, \nu > 0$, $F \in C(\mathbb{R}, \mathbb{R})$ and $f \in C(\mathbb{R}, \ell_2)$ ($f(t) := (f_i(t))_{i\in\mathbb{Z}}$ for any $t \in \mathbb{R}$) is an almost periodic function.

Theorem 1. Suppose that the following conditions hold:

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1. the function $f \in C(\mathbb{R}, l_2)$ is almost periodic;

2. the function F possesses the following properties:

   (a) it is locally Lipschitzian;

   (b) there exists a positive number α such that $F(s)s \leq -\alpha s^2$ for any $s \in \mathbb{R}$;

   (c) the function F is monotone [1], i.e., there exists a positive number β such that $(F(x_1)-F(x_2))(x_1-x_2) \leq -\beta|x_1-x_2|^2$ for any $x_1, x_2 \in \mathbb{R}$.

Then the equation (1) has a unique almost periodic solution.

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QUARTIC DIFFERENTIAL SYSTEMS WITH A CENTER-FOCUS CRITICAL POINT AND TWO PARALLEL COMPLEX INVARIANT STRAIGHT LINES OF MAXIMAL MULTIPLICITY

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Consider the real quartic system of differential equations

$$\begin{cases} \dot{x} = y + p_2(x,y) + p_3(x,y) + p_4(x,y) \equiv p(x,y), \\ \dot{y} = -(x + q_2(x,y) + q_3(x,y) + q_4(x,y)) \equiv q(x,y), \\ \gcd(p,q) = 1,\ yp_4(x,y) + xq_4(x,y) \not\equiv 0, \end{cases} \tag{1}$$

where $p_i(x,y) = \sum_{j=0}^{i} a_{i-j,j}x^{i-j}y^j$, $q_i(x,y) = \sum_{j=0}^{i} b_{i-j,j}x^{i-j}y^j$, $i = 2,3,4$ are homogeneous polynomials in x and y of degree i with real coefficients. The origin $O(0,0)$ is a singular point of either a focus or a center type for (1).

In this work, we determine the maximal multiplicity of a complex invariant straight line in the case where system (1) has two parallel complex invariant straight lines. Suppose that the quartic system (1) has two such lines, denoted by l_1 and $l_2 = \overline{l_1}$. These lines have the same multiplicity. Without loss of generality, we may assume that they are described by the equations $l_1 = x-\alpha-i$ and $l_2 = x-\alpha+i$. Thus, the system (1) takes the form:

$$\begin{cases} \dot{x} = ((x-\alpha)^2+1)(y+(1+\alpha^2)(a_{40}x^2+a_{31}xy+a_{22}y^2))/(1+\alpha^2), \\ \dot{y} = -(x+b_{20}x^2+b_{11}xy+b_{02}y^2+b_{30}x^3+b_{21}x^2y+b_{12}xy^2+b_{03}y^3+ \\ \quad +b_{40}x^4+b_{31}x^3y+b_{22}x^2y^2+b_{13}xy^3+b_{04}y^4). \end{cases} \tag{2}$$

Theorem 1. *The maximal multiplicity of the invariant straight line $l_1 = x-\alpha-i$ is three for the quartic system* (2)*. The invariant line $l_1 = x-\alpha-i$ has maximal multiplicity for system* (2) *if and only if the coefficients of system* (2) *verify one of the following two series of conditions:*

$$\begin{gathered} a_{22}=0,\ a_{31}=-1/(\alpha(1+\alpha^2)),\ b_{02}=(-1+3\alpha^2)/(2\alpha(1+\alpha^2)), \\ b_{20}=(-2\alpha+b_{40}(1+\alpha^2)^2-2a_{40}^2\alpha(1+\alpha^2)^3)/(1+\alpha^2), \\ b_{11}=2a_{40}(1+\alpha^2),\ b_{12}=-3/(1+\alpha^2)),\ b_{22}=3/(2\alpha(1+\alpha^2)), \\ b_{30}=(1-2b_{40}\alpha(1+\alpha^2)+4a_{40}^2\alpha^2(1+\alpha^2)^2)/(1+\alpha^2), \\ b_{03}=b_{21}=b_{04}=b_{13}=0,\ b_{31}=-2a_{40}; \end{gathered} \tag{3}$$

$$\begin{gathered} a_{22}=0,\ a_{31}=-1/(\alpha(1+\alpha^2)),\ b_{02}=(-1+3\alpha^2)/(2\alpha(1+\alpha^2)), \\ b_{11}=(3+a_{40}^2\alpha^2(1+\alpha^2)^2(-3+5\alpha^2))/(a_{40}\alpha^2(-3+\alpha^2)(1+\alpha^2)), \\ b_{20}=(2-6\alpha^2-a_{40}^2\alpha^2(1+\alpha^2)^4)/(2\alpha(1+\alpha^2)), \\ b_{21}=-6(1+a_{40}^2\alpha^2(1+\alpha^2)^3)/(a_{40}\alpha(-3+\alpha^2)(1+\alpha^2)^2), \\ b_{12}=-3/(1+\alpha^2), b_{22}=3/(2\alpha(1+\alpha^2)),\ b_{03}=b_{13}=0, \\ b_{30}=(7-6\alpha^2+3\alpha^4+a_{40}^2\alpha^2(1+\alpha^2)^5)/((-3+\alpha^2)(1+\alpha^2)^2), \\ b_{31}=(3\alpha^2-1+a_{40}^2\alpha^2(1+\alpha^2)^3(5+\alpha^2))/(a_{40}\alpha^2(-3+\alpha^2)(1+\alpha^2)^3), \\ b_{40}=(2-2\alpha^2-a_{40}^2\alpha^2(1+\alpha^2)^4)/(2\alpha(-3+\alpha^2)(1+\alpha^2))),\ b_{04}=0. \end{gathered} \tag{4}$$

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THE CODIMENSION OF THE PHASE PORTRAITS FOR THE FAMILY OF DEGENERATE QUADRATIC DIFFERENTIAL SYSTEMS

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Consider the family of real planar polynomial differential systems

$$x' = p(x, y), \quad y' = q(x, y),$$

$p(x, y), q(x, y) \in \mathbb{R}[x, y]$, $n = \max\{\deg(p), \deg(q)\}$; $n = 2$ - quadratic systems.

In [1] the authors gave the set of all global geometric configurations of singularities for the whole family of quadratic systems including systems in **QSD** (in total 1764 cases).

In [2] the authors grouped the set of 1764 geometric configurations into 208 global topologically distinct configurations of singularities.

The class of degenerate quadratic systems (we denote this class by **QSD**) is in the set of the most restrictive systems and has not yet being completely classified. Here we focus on the global topological classification of this family.

This paper is part of a project (initiated by Prof. J.C. Artes) whose ultimate goal is the complete classification of all topologically distinct phase portraits of quadratic systems modulo limit cycles. We also provide a label for each phase portrait inside the global codification related to the global configurations of singularities and their topological codimensions.

Our main result is:

Main Theorem. *The following statements hold:*

(i) *The family* **QSD** *possesses a total of 41 topologically distinct phase portraits and for each one of these phase portraits its codimension is determined.*

(ii) *The topological classification is done using algebraic invariant polynomials and hence it is independent of the normal forms in which the systems may be presented.*

(iii) *The bifurcation diagram of the phase portraits of systems in the family* **QSD** *is done in the twelve-dimensional parameter space* $\mathbb{R}^{12}$. *This diagram gives us an algorithm to determine for any given system its corresponding phase portrait.*

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SECTION 3: APPLIED MATHEMATICS

PARALLEL ALGORITHMS IN ENCRYPTION PROBLEMS

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The global number of compromised user accounts increased dramatically, rising from approximately 730 million in 2023 to over 5.5 billion in 2024, highlighting a dramatic escalation in cybersecurity threats and reinforcing the urgent need for robust and scalable data encryption solutions. In response to this increasing cybersecurity challenge, this work presents a parallelized implementation of the Advanced Encryption Standard (AES), specifically tailored for execution on a high-performance computing (HPC) cluster using the Message Passing Interface (MPI).

The implementation adopts a data-level parallelization strategy, where large input files, such as password databases, are divided into segments that are processed independently by multiple computing units. This approach takes advantage of the block-oriented structure of AES, allowing each process to encrypt or decrypt a portion of the data autonomously, without requiring synchronization or inter-process communication.

AES is grounded in strong algebraic principles, operating over the finite field $GF(2^8)$. Its internal structure relies on operations such as byte substitution through multiplicative inversion in Galois fields, linear transformations, and affine mappings, which together provide both confusion and diffusion, two critical properties for cryptographic strength. These mathematical constructs ensure resistance against known cryptanalytic attacks and form the basis for rigorous security analysis.

By parallelizing a mathematically rigorous algorithm such as AES across a distributed architecture, a scalable and secure solution is achieved for addressing modern data protection challenges. The resulting implementation is particularly well-suited for applications requiring high-assurance encryption of large datasets, including those in enterprise systems, cloud platforms, and scientific computing environments.

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B-SPLINE COLLOCATION METHOD FOR SOLVING SINGULAR INTEGRAL EQUATIONS WITH PIECEWISE CONTINUOUS COEFFICIENTS

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The present study is devoted to the development of an efficient computational scheme for solving Cauchy singular integral equations defined on a closed and smooth contour Γ in the complex plane. The coefficients and the right-hand side of the equation are piecewise continuous functions, given numerically on a finite set of points along the contour.

The approximate solution is constructed as a linear combination of B-spline functions and Heaviside functions [1], with the corresponding coefficients determined using the collocation method. An efficient algorithm is proposed for approximating the Cauchy singular integrals that arise in the computational scheme. The method generates a sequence of approximations that converge almost uniformly to the exact solution of the equation.

In the case where the coefficients and the right-hand side satisfy the Hölder condition with exponent $\mu \in (0,1)$, except possibly at a finite number of discontinuity points of the first kind, $t_1, ..., t_n$, sufficient conditions are established for the convergence of the proposed method. The convergence is analyzed in the norm of a Banach space $H_\mu^0(\Gamma, \rho)$ [2], consisting of functions ψ that satisfy the condition $\rho\psi \in H_\mu^0(\Gamma, t_1, ..., t_n)$, where $\rho(t) = \prod_{k=1}^n |t - t_k|^{\alpha_k}$, $\mu < \alpha_k < \mu + 1$, $k = 1, ..., n$. The subspace $H_\mu^0(\Gamma, t_1, ..., t_n)$ consists of functions that satisfy the Hölder condition with exponent μ and vanish at the points $t_1, ..., t_n$.

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MULTIDIMENSION RIEMANN METRICS TO INTEGRATING OF THE NAVIER-STOKES EQUATIONS

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Theorem 1. *The metric of the form*

$$ds^2 = 2\,dx^2 + 2\,dxdy + 2\,dxdu + 2\,dy^2 + 2\,dydz + 2\,dydv + 2\,dz^2 +$$

$$+2\,dzdw + 2\,dtdp + 2\,d\eta d\xi + 2\,d\rho d\chi + 2\,dmdn +$$

$$+Adt^2 + Bd\eta^2 + Cd\rho^2 + Edm^2, \tag{1}$$

where

$$A = 2 - U(x,y,z,t)\,u - V(x,y,z,t)\,v - W(x,y,z,t)\,w,$$

$$B = \left(-UW + \mu\frac{\partial}{\partial z}U\right)w + \left(-UV + \mu\frac{\partial}{\partial y}U\right)v + \left(\mu\frac{\partial}{\partial x}U - (U)^2 - P\right)u - Up,$$

$$C = \left(-VW + \mu\frac{\partial}{\partial z}V\right)w + \left(\mu\frac{\partial}{\partial y}V - (V)^2 - P\right)v + \left(-UV + \mu\frac{\partial}{\partial x}V\right)u - Vp,$$

$$E = \left(-\mu\frac{\partial}{\partial x}U - \mu\frac{\partial}{\partial y}V - (W)^2 - P\right)w + \left(-VW + \mu\frac{\partial}{\partial y}W\right)v +$$

$$+\left(-UW + \mu\frac{\partial}{\partial x}W\right)u - Wp$$

is the Ricci-flat metric on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{U} + \left(\vec{U}\cdot\vec{\nabla}\right)\vec{U} - \mu\Delta\vec{U} + \vec{\nabla}P = 0, \qquad \vec{\nabla}\cdot\vec{U} = 0, \tag{2}$$

where $\vec{U} = [U(\vec{x},t), V(\vec{x},t), W(\vec{x},t)]$ is the fluid velocity, $P = P(\vec{x},t)$ is the pressure, μ is the viscosity and $\vec{x} = (x,y,z)$.

Proposition. *The metrics (1) and*

$$y^2 {d_x}^2 + 2\left(l(x,z)\,y^2 + m(x,z)\right)d_x d_z + 2\,d_y d_z + \tag{3}$$

$$+\left(\left(l(x,z)\right)^2 y^2 - 2\left(\frac{\partial}{\partial x}l(x,z)\right)y + 2\,l(x,z)\,m(x,z) + 2\,l(x,z)\right){d_z}^2$$

[1] *Speaking author*: V.S. Dryuma

that are applied to integration of the KdV-equation: $\frac{\partial}{\partial z} l(x,z) - 3\, l(x,z)\, \frac{\partial}{\partial x} l(x,z) + \frac{\partial^3}{\partial x^3} l(x,z) = 0$, the KP-equation: $\frac{\partial^2}{\partial x \partial t} n(x,y,t) - 3\left(\frac{\partial}{\partial x} n(x,y,t)\right)^2 - n(x,y,t)\, \frac{\partial^2}{\partial x^2} n(x,y,t) + \frac{\partial^4}{\partial x^4} n(x,y,t) - \alpha^2 \frac{\partial^2}{\partial y^2} n(x,y,t) = 0$, the Schrödinger equation:

$$i\frac{\partial}{\partial t}\phi(x,y,t) - \kappa\frac{\partial^2}{\partial x^2}\phi(x,y,t) - K(x,y,t)\,\phi(x,y,t) = 0,$$

may be used to construct an examples of exact solutions of the system (2).

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NON-STATIONARY DIFFUSION PROBLEM WITH NON-ZERO ROBIN CONDITIONS IN THE DOUBLY PERFORATED DOMAIN

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The paper presents the homogenization of a non-stationary diffusion problem on a doubly perforated periodic domain using the multiple scales method. To demonstrate convergence in the homogenization process, the triple scale convergence method is applied. The novelty lies in the homogenization of a non-stationary diffusion problem with non-zero Robin conditions on the boundary of the small holes.

The statement of the problem

We consider the non-stationary diffusion problem in the double periodic perforated domain Ω_ε:

$$\begin{cases} \frac{\partial u_\varepsilon}{\partial t} - \operatorname{div}\left(A_\varepsilon(x)\nabla u_\varepsilon\right) = f_\varepsilon \text{ in } \Omega_\varepsilon \times (0,T) \\ \left(A_\varepsilon \nabla u_\varepsilon\right)\cdot \overline{n}_\varepsilon = 0 \text{ on } S_\varepsilon \times (0,T) \\ \left(A_\varepsilon \nabla u_\varepsilon\right)\cdot \overline{n}_\varepsilon + \alpha_\varepsilon u_\varepsilon = \varepsilon^2 h_\varepsilon \text{ on } \Sigma_\varepsilon \times (0,T) \\ u_\varepsilon = 0 \text{ on } \partial\Omega_\varepsilon \cap \partial\Omega \\ u_\varepsilon(x,0) = g_\varepsilon(x) \text{ in } \Omega_\varepsilon \end{cases} \tag{1}$$

[1] *Speaking author*: M. Dumitrache

where problem data (1) satisfies the conditions:

i) $A_\varepsilon(x) = A\left(\frac{x}{\varepsilon^2}\right)$ where $A \in \left(L^\infty_{\text{per}}(Z)\right)^3$ is an elliptical matrix, so: $A(z) = (a_{ij}(z))_{1\leqslant i,j\leqslant 3}$ has the properties:

$$\begin{cases} . \quad a_{ij} \in L^\infty_{\text{per}}(Z) \\ .. \quad \text{there exists } 0 < m < M \text{ such that} \\ m\xi_i\xi_j \leq a_{ij}(z)\xi_i\xi_j \leq M\xi_i\xi_j \text{ for any } \xi \in R^3 \text{ a.p.t. } z \in Z. \end{cases}$$

ii) $f_\varepsilon(x,t) = f\left(x, \frac{x}{\varepsilon}, \frac{x}{\varepsilon^2}, t\right)$ where $f \in L^2(\Omega \times Y^* \times Z^* \times (0,T))$

iii) $\alpha_\varepsilon(x) = \alpha\left(\frac{x}{\varepsilon^2}\right)$ with $\alpha \in L^2(\Sigma)$ and $\int\limits_\Sigma \alpha(z)\, d\sigma(z) = 0$.

iv) $h_\varepsilon(x,t) = h\left(\frac{x}{\varepsilon^2}, t\right)$ where $h \in L^2(\Sigma \times (0,T))$ and $M_\Sigma(h) \neq 0$.

v) $g_\varepsilon \in L^2(\Omega_\varepsilon)$ and $\widetilde{g}_\varepsilon \rightharpoonup g(x)$ week in $L^2(\Omega)$.

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INFORMATIONAL EXTENSIONS IN MIXED-STRATEGY MATRIX GAMES

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The information impact in game theory refers to how the availability, quality, and asymmetry of information among players influences their strategies, outcomes, and equilibria in strategic interactions. It is one of the most fundamental aspects that determine the structure and dynamics of a game. Types of Information in Game Theory:

Complete vs. Incomplete Information. In Complete information all players know the rules of the game, the possible actions, and the payoffs of all players.

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In Incomplete information at least one player is uncertain about some elements of the game (e. g., payoffs, available strategies, or the "type" of another player).

Perfect vs. Imperfect Information. In Perfect information every player knows all past actions when making a decision and in Imperfect information some actions or states are hidden from players.

When players are uncertain, they may randomize their actions to stay unpredictable. Mixed strategies are often used in games with imperfect information to avoid being exploited. A mixed strategy in game theory involves a player choosing an action randomly from their available strategies, with each strategy having a certain probability of being selected. So we consider the matrix game in the following strategic form: $\Gamma = \langle X, Y, XAY^T \rangle$ where $A = \|a_{ij}\|_{i=\overline{1,n}}^{j=\overline{1,m}}$, $X = \left\{ x \in R^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1 \right\}$ and $Y = \left\{ y \in R^m : y_j \geq 0, \sum_{i=1}^{m} x_j = 1 \right\}$. All players know exactly the payoff matrices A and the sets of mixed strategies. Players maximize their payoffs. Consider the following "informational decision making model": the player 1 knows exactly the value of the mixed strategy chosen by the player 2. So we will analyze the game in complete information (the players know exactly the normal form of the game) and perfect information over the sets of the mixed strategies of the player's 2. The conditions described above stipulate that we can use the set of informational extended strategies of the player 1 which are presented like this $M(y) = \left\{ m(y) = \|m_{ij}(y)\|_{i\in I}^{j\in J} \right\}$ where $m_{ij}(y)$ is the probability of choosing row i by player 1 when it is known that player 2 chooses column j with probability y_j. It should be mentioned that the player 2 do not know the informational extended strategies of the player 1 and from this point of view we can consider that the game is in imperfect information structure over the sets of the informational extended strategies of the player 1.

For player 1 the $\sum_{i\in I} a_{ij} m_{ij} y_i$ is the average value of the payoff's when, knowing the value y_j of player 2's mixed strategy, he will choose the informational extended mixed strategy $m_{.j}$. So we can construct the set of games $\Gamma(y) = \left\langle M(y), Y, \sum_{j\in J} \sum_{i\in I} a_{ij} m_{ij} y_i \right\rangle$. Each of these games can be solved using the simplex method of linear programming.

A STOCHASTIC MODEL OF SURFACE SOIL POLLUTION FROM A STATIONARY SOURCE

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We present a stochastic model of surface soil pollution from a stationary source based on the symmetric stochastic motion at finite speed in the plane $\mathbb{R}^2$, also called the planar Markov random flight [1,2,3], whose lifetime is a random variable with given distribution. This means that the pollution process is carrying out by some particles that are emitting from the source and make chaotic finite-velocity stochastic motions until they settle on the ground. The time interval between the emission of a particle and the moment it falls on the soil is a positive random variable called the particle's lifetime.

We consider a heavy-particle model, in which the lifetime is supposed to be an exponentially-distributed random variable, and obtain an explicit formula for the stationary probability density of the pollution process expressed in terms of McDonald functions with variable indices.

We also study a light-particle model, in which the lifetime is a gamma-distributed random variable. In this case, the stationary probability density of the pollution process is given in the form of a definite integral calculated numerically, as well as in the form of a functional series composed of the hypergeometric functions with variable coefficients.

These stationary densities are plotted in a figure and numerically calculated tables that demonstrate the behaviour of the pollution process on long time intervals. These calculations show that, for large values of time, the stationary density of soil pollution in the heavy-particle model is a nonlinear monotonously decreasing function, as the distance from the source increases, taking its maximal value in the neighbourhood of the source. In the light-particle model, the maximal value of the stationary density is taken at some distance from the source.

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HIDDEN MARKOV MODELS WITH FINAL SEQUENCES OF OBSERVABLE STATES

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Let consider a hidden Markov model (HMM), represented by underlying Markov process L with finite sets of hidden states $H = \{h_1, h_2, \ldots, h_\varpi\}$ and observable states $V = \{v_1, v_2, \ldots, v_\omega\}$. The system L starts from each state $h \in H$ with initial probability $p^*(h)$. At each moment of time $t = 0, 1, 2, \ldots$, the system L passes from a state $g \in H$ to the next state $h \in H$ with the probability $p(g, h)$ and emits each observable state $v \in V$ with the probability $q(h, v)$.

Initially, we study the HMM with a single final sequence of observable states $U = (u_1, u_2, \ldots, u_m) \in V^m$. The amount of time T, needed to observe U, represents the evolution time and can be analyzed by investigating the joint stochastic system J over the set of states $H \times V$. The same strategy from [1], based on Knuth-Morris-Pratt algorithm ([3]), is used. Each state $(h, v) \in H \times V$ is enriched with a number k, $0 \le k \le m$, that represents the length of the longest observed prefix from the final sequence of states. The set of all possible states is $Z = \{(h, v, 0) \mid h \in H,\ v \in V \backslash \{u_1\}\} \cup \{(h, u_k, k) \mid h \in H,\ 1 \le k \le m\}$. As result, J becomes a Markov process with $\varpi(\omega - 1 + m)$ states and ϖ final sequences of states: $\{((h, u_m, m)) \mid h \in H\}$. Based on [2], the distribution of T evolves into a homogeneous linear recurrence (HLR) of order $\varpi(\omega + m)$.

Next, we consider the HMM with multiple final sequences of observable states $U^{(\ell)} = (u_1^{(\ell)}, \ldots, u_m^{(\ell)}) \in V^m, \ell = \overline{1, r}$. The approach based on Aho-Corasick algorithm ([4]), studied in [1], can be applied. Each state $(h, v) \in H \times V$ is enriched with a number k, $0 \le k \le m$, that represents the length of the longest observed prefix from the final sequences of states, and the corresponding prefix $Y_k^{(\ell)} = (u_1^{(\ell)}, \ldots, u_k^{(\ell)})$, $\ell \in \{0, 1, \ldots, r\}$. The set of all possible states is

$$Z = \bigcup_{h \in H} \left(\{(h, v, Y_0^{(0)}, 0) \mid v \in V \backslash U_1\} \cup \{(h, u_k^{(\ell)}, Y_k^{(\ell)}, k) \mid k = \overline{1, m},\ \ell = \overline{1, r}\} \right)$$

So, J shifts to a Markov process with maximum $\varpi[\omega + r(m-1)]$ states and $r\varpi$ final sequences of states: $\{((h, u_m^{(\ell)}, U^{(\ell)}, m)) \mid h \in H,\ \ell = \overline{1, r}\}$. From here and [2], the distribution of the evolution time T is converted to a HLR, whose order does not exceed $\varpi(\omega + mr)$. As result, the algorithms presented in [2] can be applied for determining the main probabilistic characteristics of T.

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STATIONARY EQUILIBRIA FOR DYNAMIC POSITIONAL GAMES ON GRAPHS

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We consider a class of m-player dynamic positional games with average and discounted payoffs on graphs that extends the two-player zero-sum mean payoff games on graphs from [1-3]. In [1-3] are proven the existence of the value and the optimal pure stationary strategies for the antagonistic positional games with average and discounted payoffs for the players. In this contribution we extend these results for m-player positional games with average and discounted payoffs on finite directed graphs and propose new approaches for determining the optimal stationary strategies for the players. We show that for an arbitrary m-player dynamic positional game with average payoffs on a finite directed graph there exists a Nash equilibrium in mixed stationary strategies and for an arbitrary two-player zero-sum average positional game there exists a Nash equilibrium in pure stationary strategies. Additionally we show that for an arbitrary dynamic positional game with discounted payoffs on a finite directed graph there exists a Nash equilibrium in pure stationary strategies. Some approaches for determining Nash equilibria in pure and mixed stationary strategies for the considered class of dynamic positional games on graphs are proposed. A detailed information related to applications of the considered class of games can be found in [4].

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NON-MARKOVIAN REAL-TIME MODELING OF AN EMERGING INFECTIOUS DISEASE: A FRENCH RETROSPECTIVE ON COVID-19 MATHEMATICAL PROJECTIONS

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SARS-CoV-2 virus has spread over the world rapidly creating one of the largest pandemic ever. The absence of immunity, presymptomatic transmission, and the relatively high level of virulence of the COVID-19 infection led to a massive flow of patients in critical care units (CCU). This unprecedented situation called for rapid and accurate mathematical models to best inform public health policies.

Here, we introduce an original discrete-time model – developped in March 2020 and gradually improved since – that combines the computational and tractability benefits of deterministic systems and the short-term accuracy of non-Markovian dynamics, taking into account the effect of age of infection on the natural history of the disease [1]. By analysing the hospital time series of COVID-19 in France, we were able to provide early estimates of the main epidemiological parameters with only limited publicly available data. Since then, the model provided an everyday robust framework used to nowcast the epidemic in France, investigate counterfactual scenarios of public health interventions and anticipate CCU overload, while gradually updating hospital care

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kinetics, variant of concern dynamics and vaccine rollout. The model translated into an online forecasting application and several published projections and support to decision-making at both local and national scale [2]. In the light of five years since the start of the pandemic, through failures and successes, we review the real-time improvement history and compare the projections to alternative phenomenological/statistical forecasting approaches [3]. Finally, we discuss the both the methodological and sanitary implications of accounting for non-Markovian dynamics and the modelling [4] design of immune waning [5].

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FIRST-PASSAGE PROBLEMS FOR A DIFFUSION PROCESS USED IN POPULATION GENETICS

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We consider the diffusion process $\{X(t), t \geq 0\}$ defined in the interval $[0, 1]$ by the stochastic differential equation

$$\mathrm{d}X(t) = \{X(t)[1 - X(t)]\}^{1/2}\mathrm{d}B(t), \tag{1}$$

where $\{B(t), t \geq 0\}$ is a standard Brownian motion. This process is a particular Wright-Fisher process, which is important in population genetics and also in mathematical finance. We define the *first-passage time*

$$\tau(x) = \inf\{t > 0 : X(t) = 0 \text{ or } 1 \mid X(0) = x \in (0, 1)\}. \tag{2}$$

In this talk, we will first compute the expected value of the area covered by $\{X(t), t \geq 0\}$ in the interval $[0, \tau(x)]$. We will also solve an optimal control problem for

$$\mathrm{d}X_u(t) = b[X_u(t)]\,u[X_u(t)]\,\mathrm{d}t + \{X_u(t)[1 - X_u(t)]\}^{1/2}\mathrm{d}B(t), \tag{3}$$

where $b(\cdot) \neq 0$, and $u(\cdot)$ is the control variable, which is assumed to be a continuous function.

Finally, we will add jumps according to a Poisson process to $\{X(t), t \geq 0\}$ that are uniformly distributed on the interval $[-1, 1]$, and we will compute the probability that the jump-diffusion process will take a value smaller than or equal to 0 before a value greater than or equal to 1.

MATHEMATICAL AND COMPUTATIONAL FRAMEWORK FOR WAVE–NEURAL MODELING

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This work develops a unified mathematical and computational framework for modeling two-dimensional wave propagation and training a neural network to classify wave phenomena. We start from the 2D wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\Big(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\Big),$$

with appropriate initial and Dirichlet boundary conditions. An explicit finite-difference time-domain (FDTD) scheme is implemented:

$$u_{i,j}^{n+1} = 2u_{i,j}^n - u_{i,j}^{n-1} + \left(\frac{c\,\Delta t}{\Delta x}\right)^2 \big(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n\big) + \left(\frac{c\,\Delta t}{\Delta y}\right)^2 \big(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n\big),$$

subject to

$$u_{i,j}^n = 0 \quad \text{on barrier cells}, \quad \left(\frac{c\,\Delta t}{\Delta x}\right)^2 + \left(\frac{c\,\Delta t}{\Delta y}\right)^2 \leq 1.$$

This captures reflection and diffraction at obstacles.

All simulations run in parallel on a CPU-only HPC cluster. The wavefield snapshots generated over time serve as training images. Two classes are defined: "Normal_Wave" (unobstructed propagation) and "Diffraction" (wavefront bending around barriers).

The neural model is a convolutional network with two convolutional layers (3×3 kernels, ReLU activation, max-pooling) followed by two fully connected layers. A neuron computes

$$z^{(\ell)} = W^{(\ell)} a^{(\ell-1)} + b^{(\ell)}, \quad a^{(\ell)} = \sigma(z^{(\ell)}),$$

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and final logits pass through sigmoid to produce a single output $\hat{y} \in (0,1)$. Training minimizes the binary cross-entropy:

$$J(W,b) \;=\; -\frac{1}{m}\sum_{i=1}^{m}\Big[\,y^{(i)}\log\big(\hat{y}^{(i)}\big) + \big(1-y^{(i)}\big)\log\big(1-\hat{y}^{(i)}\big)\Big],$$

with gradients synchronized across processes via MPI `Allreduce`. The model converges in under 20 epochs.

Validation on hold-out data yields 96.5% accuracy and F1-scores above 0.95 for both classes, demonstrating robust recognition of diffraction patterns even under noisy or novel obstacle configurations.

VALUATION OF OPTION CONTRACTS: THE DISCRETE CASE

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The valuation of option contracts plays a crucial role in modern financial markets, particularly for managing risk and designing investment strategies. This study focuses on the discrete-time valuation of European and American option contracts, including both call and put variants. While continuous-time models such as the Black-Scholes-Merton framework have been extensively used in theoretical finance, discrete-time methods remain essential for practical applications due to their numerical flexibility and ability to handle early exercise features inherent in American options.

We examine the binomial and trinomial tree models as foundational tools for valuing options in discrete time. The binomial model, introduced by Cox, Ross, and Rubinstein [1], provides an intuitive and computationally efficient method for approximating option prices by modeling the evolution of the underlying asset price through a recombining tree structure. European options [2], which can only be exercised at expiration, are straightforward to evaluate using this model via backward induction. American options [3], by contrast, require the consideration of early exercise at each node, making their valuation more complex and computationally intensive. The optimal stopping rule is determined by comparing the intrinsic value with the expected discounted continuation value at each step.

We further explore enhancements such as the trinomial model, which offers improved accuracy and convergence speed, and lattice-based methods that enable the valuation of exotic options and accommodate variable interest rates or

dividends [4]. Our numerical results confirm the convergence of discrete models to their continuous counterparts and demonstrate the impact of volatility, time steps, and interest rates on pricing accuracy. The analysis also highlights the distinction between the fair prices of European and American options under various market conditions. This paper underscores the continued relevance of discrete-time valuation in financial engineering. It also serves as a foundational reference for further development of finite-difference and Monte Carlo methods in option pricing.

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CONSTRUCTIVE INTEGRAL REPRESENTATIONS OF BROWNIAN FUNCTIONALS

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The stochastic integral representation or, more generally, the martingale representation theorem is a fundamental result of stochastic calculus. It is widely used in option pricing and risk management models, in the structural analysis of martingales and their relationship to Brownian motion, and in solving problems in control theory and filtering. By showing how every square-integrable martingale driven by Brownian motion can be expressed as a stochastic integral, the theorem provides a unified language for modeling and manipulating random phenomena. Its applicability therefore extends far beyond finance into physics, engineering, economics, and any discipline that grapples with uncertainty. The foundational result on stochastic integral representations is the Clark–Ocone formula established by Ocone ([1]). However, that formula breaks down for functionals that are not Malliavin-smooth. Glonti and Purtukhia ([2]) extended

Clark–Ocone to cover the situation where the functional itself may be Malliavin-nonsmooth, provided its conditional expectation is Malliavin-differentiable, and they gave a way to construct the corresponding integrand. Here we propose a new constructive approach that yields stochastic integral representations for both Malliavin-smooth and Malliavin-nonsmooth functionals.

Theorem 1. If $G(S,T) = I_{\{\max_{u\in[S,T]} B_u \le c\}}$, then the function $V(t,x) = E[G(t,T)|B_t = x]$ satisfies the requirements of the Ito's formula (i.e. $V(\cdot,\cdot) \in C^{1,2}([0,T]\times R)$) and the following stochastic integral representation is valid

$$G(0,T) = 2\Phi\Big(\frac{c}{\sqrt{T}}\Big) - \int_0^T G(0,t)\cdot\frac{1}{\sqrt{T-t}}\varphi\Big(\frac{c-B_t}{\sqrt{T-t}}\Big)dB_t \quad (P-a.s.),$$

where Φ is the standard normal distribution function and φ is its density.

Theorem 2. If $F(T) = \exp\{I_{\{B_T\le c\}}\}$, then the function $V(t,x) = E[F(T)|B_t = x]$ satisfies the requirements of the Ito's formula and we have

$$F(T) = 1 + (e-1)\Phi\Big(\frac{c}{\sqrt{T}}\Big) - \int_0^T \frac{e-1}{\sqrt{T-t}}\varphi\Big(\frac{c-B_t}{\sqrt{T-t}}\Big)dB_t \quad (P-a.s.),$$

where e is the Euler number.

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MODELING THE FORMATION OF ONE-DIMENSIONAL FLOWS OF A REAL GAS AT LARGE STROUHAL NUMBERS

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This paper presents a study of the formation of one-dimensional flows of a real gas, where a real gas is considered an ideal gas, whose processes are affected by thermal conductivity, viscosity and other physical effects [1]. The formation of the flow field is analyzed using the example of solving the problem of the interaction of a viscous heat-conducting gas with the Clapeyron equation of state and a heat-conducting wall with different initial temperatures. A characteristic feature of the resulting flows is due to the development in the gas medium

of narrow dissipative zones with large parameter gradients. Using the motion scheme of an ideal gas described by the Euler equations, it is not possible to determine heat fluxes, to clarify the development and influence of dissipative zones on the overall flow pattern. Therefore, the study of flows of a real gas is carried out based on the Navier-Stokes equations, which effectively model the laws of conservation of mass, momentum and energy [1]. The wall temperature distribution is described by a linear heat conductivity equation. As a result, we obtain a system of equations that determines, for given initial and boundary conditions, the parameters as functions of time and spatial coordinate.

When solving problems with a discontinuity of the initial data in the vicinity of the initial moment of time, a feature in the behavior of the solution arises. Therefore, analyzing the formation of flows at small times is of particular interest. We introduce characteristic scales for the parameters and pass in the equations and conditions to dimensionless quantities. In this case, for the resulting dimensionless complexes of characteristic scales, we obtain the Strouhal number Sh, the Reynolds number Re, etc. In the initial phase of flow development, the characteristic times are small, and the Strouhal number Sh is large in the entire flow region. Consequently, the initial phase of flow development can be studied by expanding the parameters into series in a small parameter equal to the inverse Strouhal number Sh-1. In this paper, expressions for the zero leading approximation of the parameters were obtained. Further solution refinement is carried out similarly, determining the next terms in the expansions of the parameters.

The solutions, constructed in the form of uniformly suitable series in a small parameter, reveal the mechanism of the origin of flows in problems with a discontinuity of the initial data, and are of interest for subsequent use in the numerical solving of nonlinear problems.

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WEAK SINGULAR INTEGRAL OPERATORS AND THEIR ESTIMATIONS IN HÖLDER SPACES

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Algorithmic implementation of collocation, quadratures, splin-collocation and splin-quadratures methods for solving weak-singular integral equations (WSIE) of second kind leads to necessity of concrete evaluation of Fredholm and Volterra weakly-singular operators and their modifications, appearing in algorithm application. We mention that in works [1] and [2] only the existence of majority of indicated constants for weak-singular integral (WSIO) is proved. This fact does not permit their application to numerical algorithms.

In this work concrete values of constants majoring Fredholm and Volterra WSIO are estimated depending on their modifications appearing in algorithms of directly-approximate methods in solving of integral equations of second kind given on a segment.

Denote by $H_\alpha[a,b]$ the Hölder space with exponent α $(0<\alpha<1)$ on $[a,b]$.

Theorem 1. *Let the function* $h(t,s)\in H_{\alpha,\alpha}[a,b]$ *and* $0<\gamma<1$. *Then, for any function* $\varphi(t)\in C[a,b]$, *we have*

$$G(t)=\int_a^b \frac{h(t,s)}{|t-s|^\gamma}\varphi(s)ds\in H_\theta[a,b],\ \theta=min(\alpha;1-\gamma),$$

with Hölder's constant $c_1=\|\varphi\|_c\Big(\|h\|_c\dfrac{2(2^\gamma+1)}{1-\gamma}+H^t(h;\alpha)\dfrac{2(b-a)^{1-\gamma}}{1-\gamma}\Big)$.

Let us define the next functions:

$$K_\rho(t,s)=\begin{cases}\dfrac{h(t,s)}{|t-s|^\gamma},0<\gamma<1; & |t-s|\geq\rho,\\ \dfrac{h(t,s)}{\rho^\gamma}; & |t-s|<\rho,\\ \rho<<\min\{b-a;1\}.\end{cases}$$

For Fredholm WSIO, we prove

Theorem 2. *Assume that* $h(t,s)\in H_{\alpha,\alpha}[a,b]$; $\psi(t)\in C_\alpha[a,b]$; $0<\gamma<1$. *Then functions* $\chi_\rho(t)=\int\limits_a^b[\frac{h(t,s)}{|t-s|^\gamma}-K_\rho(t,s)]\cdot\psi(s)ds$, $\eta_\rho(t)=\int\limits_a^b K_\rho(t,s)\cdot\psi(s)ds$ *verify the relations:*

$$|\chi_\rho(t)|\leq c_4\rho^{1-\gamma}\|\psi\|_c,\quad c_4=\frac{2\gamma}{1-\gamma}\|h\|_c;$$

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$$|\eta_\rho(t)| \le (c_5 - c_4\rho^{1-\gamma})\|\psi\|_c, \quad c_5 = \frac{2(b-a)^{1-\gamma}}{1-\gamma}\|h\|_c;$$

$$\chi_\rho(t), \eta_\rho(t) \in H_\theta[a,b], \quad \theta = \min(\alpha; 1-\gamma).$$

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ANALYTICAL LOWER BOUNDS FOR CONDITIONAL LABEL PROBABILITY CONFIDENCE INTERVALS

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In the study of valid *distribution-free* confidence intervals for the conditional label probability $\pi_P(x) = \mathbb{P}\{Y = 1 \mid X = x\}$, when $(X_i, Y_i) \overset{\text{iid}}{\sim} P$ with $Y_i \in \{0, 1\}$, one can guarantee that:

$$\mathbb{P}\big\{\pi_P(X_{n+1}) \in \hat{C}_n(X_{n+1})\big\} \ge 1 - \alpha \quad \text{for } \textit{every} \text{ distribution } P$$

Barber et al. shows that unlike predictive sets for Y (e.g. conformal prediction), the length of any such interval $\hat{C}_n(x) \subseteq [0,1]$ cannot vanish as $n \to \infty$. Define the class of functions

$$\mathcal{F}_{t,a} = \big\{ f : [0,1] \to [0,1] : \mathbb{E}[f(Z)] \ge 1 - a \ \ \forall\, Z \in [0,1] \text{ with } \mathbb{E}[Z] = t \big\}$$

One shows in closed form that $\ell(t,a) = \inf_{f \in \mathcal{F}_{t,a}} \int_0^1 f(s)\, ds$ where $\ell(t,a)$ is defined as:

$$\ell(t,a) = \begin{cases} 2(1-a)\,t, & a \ge \frac{1}{2}, \\ \dfrac{t}{2a}, & a \ge t \text{ and } 0 < a < \frac{1}{2}, \\ 1 - \dfrac{a}{2t}, & a < t < \frac{1}{2}, \end{cases} \quad \ell(0,0) = 0, \quad \ell(t,a) = \ell(1-t,a) \text{ for } t > \tfrac{1}{2}.$$

If Π_P denotes the distribution of $\pi_P(X)$ under $X \sim P_X$, then any interval $\hat{C}_n$ satisfying

$$\mathbb{P}_{(X_i,Y_i)\overset{\text{iid}}{\sim}P}[\pi_P(X_{n+1}) \in \hat{C}_n(X_{n+1})] \ge 1-\alpha \quad \text{must satisfy} \quad \mathbb{E}\big[\text{leb}(\hat{C}_n(X_{n+1}))\big] \ge L_\alpha(\Pi$$

$$\text{Where } L_\alpha(\Pi_P) = \inf_{\substack{a:[0,1]\to[0,1] \text{ measurable} \\ \mathbb{E}_{T\sim\Pi_P}[a(T)]\le\alpha}} \mathbb{E}_{T\sim\Pi_P}\big[\ell\big(T, a(T)\big)\big].$$

In three illustrative link-function cases one obtains fully explicit formulas:

(i) CDF–link (uniform case). If X is any continuous distribution with CDF $F(x)$ and $T = F(X)$ then $T \sim \mathrm{Unif}[0,1]$, then

$$L_\alpha = \int_0^1 \ell(t,\alpha)\,dt = 2\int_0^{1/2} \ell(t,\alpha)\,dt = \begin{cases} 1 - \frac{3\alpha}{2} + \alpha\ln(2\alpha), & 0 < \alpha < \frac{1}{2}, \\ \dfrac{1-\alpha}{2}, & \frac{1}{2} \le \alpha \le 1. \end{cases}$$

(ii) Monomial-link $\pi(x) = x^k$ on $[0,1]$. Here $T = X^k$ has density $f_T(t) = \frac{1}{k}\,t^{1/k-1}$, and

$$L_\alpha = \frac{1}{k}\int_0^1 \ell(t,\alpha)\,t^{\frac{1}{k}-1}\,dt =$$

$$= \begin{cases} \dfrac{\alpha^{1/k}}{2(k+1)} + \big((\frac{1}{2})^{1/k} - \alpha^{1/k}\big) - \dfrac{\alpha}{2(1-k)}\Big((\frac{1}{2})^{1/k-1} - \alpha^{1/k-1}\Big), & 0 < \alpha < \frac{1}{2}, \\ \dfrac{2(1-\alpha)}{k+1}, & \frac{1}{2} \le \alpha \le 1. \end{cases}$$

(iii) Arcsine-link $\pi(x) = \sin^2\!\big(\frac{\pi x}{2}\big)$. Then $T \sim \mathrm{Arcsine}(0,1)$ with density $f_T(t) = 1/(\pi\sqrt{t(1-t)})$, and one finds

$$L_\alpha = \int_0^1 \ell(t,\alpha)\,\frac{dt}{\pi\sqrt{t(1-t)}} =$$

$$= \begin{cases} \dfrac{2}{\pi}\arcsin(\sqrt{\alpha}) - \dfrac{\sqrt{\alpha(1-\alpha)}}{\pi\,\alpha} - \dfrac{\alpha}{\pi}\ln\dfrac{1+\sqrt{1-\alpha}}{\sqrt{\alpha}}, & 0 < \alpha < \frac{1}{2}, \\ 1-\alpha, & \frac{1}{2} \le \alpha \le 1. \end{cases}$$

(iv) Monomial-link, Beta prior $X \sim \mathrm{Beta}(a,b)$, $T = X^k$ wherer $a, b, k \ge 1$. Then T has PDF $f_T(t) = \frac{1}{kB(a,b)} t^{\frac{a}{k}-1}(1-t^{\frac{1}{k}})^{b-1}$ then we find that, with $B_x(p,q) = \int_0^x u^{p-1}(1-u)^{q-1}\,du$ the standard incomplete Beta-function, and $B(p,q) = B_1(p,q)$:

$$L_\alpha = \frac{1}{2\alpha\,B(a,b)}\;B_{\alpha^{1/k}}(a+k,b) + \;\frac{B_{(1-\alpha)^{1/k}}(a,b) - B_{\alpha^{1/k}}(a,b)}{B(a,b)} \;-$$

$$\frac{\alpha}{2\,B(a,b)}\Big[B_{(1/2)^{1/k}}(a-k,b) - B_{\alpha^{1/k}}(a-k,b)\Big]$$

$$+\;\frac{1}{2\alpha\,B(a,b)}\Big[B(a,b) - B_{(1-\alpha)^{1/k}}(a,b) \;-\; B(a+k,b) + B_{(1-\alpha)^{1/k}}(a+k,b)\Big]$$

$$\text{for } 0 < \alpha \leq 1/2 \text{ and } L_\alpha = 2(1-\alpha)\frac{B(a+k,b)}{B(a,b)} \quad \text{for } 1/2 < \alpha < 1$$

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A COMPUTATIONAL COMPARISON OF MIXED-INTEGER FORMULATIONS FOR CONCAVE PIECEWISE-LINEAR FUNCTIONS

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Optimization problems that have polyhedral feasible sets and piecewise-linear objective functions can be modeled as mixed-integer programs (MIPs). In many cases, the resulting problems can be solved efficiently using modern MIP solvers [3].The classical MIP formulations for piecewise-linear functions employ a number of binary variables that is linear in the number of pieces of the function. More recently, logarithmic MIP formulations for piecewise-linear functions have been introduced, which use a number of binary variables that is logarithmic in the number of pieces of the function [3].

When the piecewise-linear functions are concave, and the objective function is being minimized, an additional formulation with a linear number of binary variables is available:

$$\begin{aligned}\min\Big\{\textstyle\sum_{i=1}^{n}\sum_{p=0}^{P}(f_i^p z_i^p + s_i^p y_i^p) \;\Big|\; Ax \leq b, x_i = \sum_{p=0}^{P} y_i^p, i \in [n],\\ 0 \leq y_i^p \leq B_i z_i^p, z_i^p \in \{0,1\}, i \in [n], p \in \{0,\ldots,P\}\Big\}.\end{aligned} \tag{1}$$

This formulation has been known since at least the 1960s. A significant advantage of formulation (1) is that it preserves the problem structure for many problems with concave piecewise-linear costs. This enables specialized algorithms for well-known discrete optimization problems to be used for solving problems with concave piecewise-linear costs [1-2].

In this work, we study the computational performance of formulation (1) when used with a general-purpose MIP solver. Specifically, we perform computational comparisons between formulation (1), and classical MIP formulations with a linear number of binary variables.

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EFFICIENT METHODS FOR SOLVING THE LINEAR MULTI-CRITERIA OPTIMIZATION MODEL IN INTEGERS

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Currently, multi-criteria optimization models are in increasing demand. This is due to the fact that numerous problems in the most diverse practical fields require solutions that are acceptable from the point of view of several criteria. A wide range of practical optimization problems in various fields lead to the solution of mult-icriteria linear optimization models [1] in integers. In this paper we propose two methods for solving the next linear multi-criteria model in integer:

$$\begin{cases} \left\{ \begin{array}{c} \min \\ \max \\ x \in D \end{array} \right\} F_k(x) = \sum_{j=1}^{n} c_{kj} x_j, \quad k = \overline{1,r} \\ A \cdot x \leq b \\ x \in Z^+ \end{cases} \tag{1}$$

The paper proposes a comparative analysis of two methods: the global utility maximization method [2] and the fuzzy technique method. Assuming that all criteria are equidistant, the fuzzy method determines the optimal compromise solution, as the closest one to the optimal solutions of each criterion, evaluating the indices:

$$\mu_k(Z^k)) = \begin{cases} 1, \text{if } Z^k(x) \leq L_k \\ \dfrac{U_k - Z^k(x)}{U_k - L_k}, \text{ if, } L_k < Z^k(x) < U_k; \\ 0, \text{ if} Z^k(x) \geq L_k \end{cases} \tag{2}$$

The efficiency of the methods can be appreciated through concrete decision-making situations. The theoretical justifications of the methods, as well as an example solved by both methods, are provided in the proposed full study.

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SECTION 4: COMPUTER SCIENCE AND IT

MATHEMATICAL MODELS AND AGGREGATION FUNCTIONS

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A separate direction in the decision-making methodology is the collective expertise. The essence of the collective expertise consists in the evaluation of a set of alternatives by a number of experts based on certain criteria. As a result, a three-dimensional array of assessments is obtained: each alternative by each expert in accordance with each criterion. Most probably these assessments carry in themselves the degree of quality of each alternative. Hence each alternative is evaluated qualitatively based on each criterion, but then a question arises – how, from the set of qualitative assessments on each criterion, to obtain an integral qualitative assessment of the alternative. One of the approaches would be the use of aggregation functions. In the most simplified case, at the input we have the assessments based on the criteria, and at the output a resulting assessment expressing the integral quality of the alternative. A class of aggregation functions would be the functions for calculating the unweighted average or weighted averages. So for the sequence of evaluations (numbers) $e_1, e_2, ..., e_{n-1}, e_n$ such an aggregation function $M = (p_1e_1 + p_2e_2 + ... + p_{n-1}e_{n-1} + p_ne_n)/(p_1 + p_2 + ... + p_{n-1} + p_n)$, where $p_1, p_2, ..., p_{n-1}, p_n$, are the necessary weights, could be used in the general case. From an empirical point of view, this is the correct approach: evaluations with small (more severe) values compensate for the action of evaluations with large values. Would there be any mathematical approach that would lead to such an aggregation function?

Find a value x that minimizes the functional:

$$F(x) = p_1(x - e_1)^2 + p_2(x - e_2)^2 + ... + p_n(x - e_n)^2 \quad (1)$$

By differentiating the function by x and solving the obtained equation, it will be deduced that the value of x is calculated by the weighted average. So if the aggregation problem is formulated accordingly to the minimization model discussed above, we will arrive at an aggregation function expressed by the weighted average.

For the formulation of the following approach, it is assumed that the sequence of evaluations is ordered in ascending order.

We should find a value for x such that the weighted sum of the differences between x and the evaluations smaller than x is equal to the weighted sum of the differences between the evaluations larger than x and x:

$$p_1(x - e_1) + ... + p_i(x - e_i) = p_{i+1}(e_{i+1} - x) + ... + p_n(e_n - x) \qquad (2)$$

From this formulation, it is observed that it is assumed that the first i evaluations are smaller than x, and the other ones are bigger. Solving the equation, we obtain that the value of x is calculated by the weighted average and it does not matter how many evaluations are smaller than x, that is, the assumed value i does not matter. Therefore, even with a rephrasing of the aggregation problem in such a way, we will also get to the weighted average.

These rationings exemplify that aggregation functions can be deduced not only empirically, but also by applying mathematical models.

HANDWRITTEN TEXTS PROCESSING: CHALLENGES AND SOLUTIONS

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The digitisation of handwritten historical texts is a difficult problem from several points of view. In addition to the difficulties created by the physical deterioration of manuscripts, which is characteristic of most ancient texts, in the case of handwritten texts, we find not only that the handwriting differs from one person to another, but even that there are differences within the same document, written by the same person. One of the major challenges for early medieval texts is continuous writing (scripta continua), which has no spaces or other separators between words and sentences. There is also usually no differentiation between upper and lower case. There is a lack of standardised spelling, and inevitable human errors occur when copying texts, etc.

The biggest problem, however, is created by the lack of high-volume resources that would allow the use of modern machine learning-based technologies.

The paper examines approaches based on both traditional neural networks and Large Language Models (LLM), which allow solving the mentioned problems even for small collections of texts of the corresponding period.

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We have applied mixed techniques such as: the creation of dictionaries of limited size with their expansion based on newly recognised documents, the use as a learning source of printed documents from a later period (17th century), which are closer to medieval handwritten ones. In particular, the corpus of texts from the 17th century has been successfully used to develop the algorithm for separating a medieval scripta continua text into words. The training was carried out based on the New Testament (1648) spaces winding, thus yielding a document with uninterrupted handwriting, but also having its equivalent in a word-spaced format. DistilBERT-based neural network served as a tool, which was adapted to solve this problem.

Character recognition techniques using classical neural networks (e.g. ABBYY FineReader) and GPT-4 models were evaluated and compared, highlighting the large training data requirements in the case of traditional networks versus the need for optimal prompt definition in the case of LLM models. It also highlighted the potential of conversational feedback in improving the recognition of individual characters when LLM models are used through the ChatGPT interface.

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ON SEMANTIC SEGMENTATION OF MOLDAVIAN DIALECT MAPS

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Historic linguistic maps, such as those from Moldovan dialectal atlases, contain semi-structured data with numbered geographical points and associated dialectal transcriptions in Cyrillic. Digitizing these maps is crucial for preserving linguistic heritage. However, standard OCR techniques struggle with such maps because the annotations (entries on the maps) are not arranged in regular lines or columns. Each map entry consists of a location number surrounded by one or more Cyrillic words or phrases, often written at various positions and orientations around the number. Key challenges in processing these dialect maps include irregular layout, which means text annotations appear above, below, or next to the point numbers, sometimes diagonally, with no consistent alignment or reading order; text may overlap with other entries or with map elements;

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each entry comprises a numeric identifier and an answer (word/phrase) forming a logical pair, but they are not distinctly separated or easily isolatable by simple image processing and others regarding OCR.

To address these challenges, we trained a YOLO-based object detector to automatically locate and classify key elements on the map. This first stage detects bounding boxes for each entry (the combination of a point number and its associated text), as well as other semantic elements like the map's legend, the map index label, and table. By treating each entry as an object, the detector learns to find text clusters regardless of placement. Detected entry regions are then cropped and passed to a second YOLO model specialized for separating the entry into its two components: (a) the point number and (b) the answer text. This secondary model was trained on 2,000 entry crops annotated for the two component classes: the textual answer and the numeric label. Next steps are to integrate OCR models for Moldovan Cyrillic, transliterate the recognized text into modern Romanian Script, link the recognized/transliterated text with its geospatial coordinates, and export the results as interoperable XML/JSON datasets suitable for regenerating digital dialect maps.

We annotated a collection of maps using the CVAT tool to create a training dataset for the detection models. Experiments with training set sizes of 30, 50, 100, and 120 annotated maps showed that larger datasets and longer training substantially improved detection performance. Our best configuration, using the YOLOv11 architecture trained on over 50 maps for 300 epochs, achieved excellent accuracy (mAP@50 > 0.95) in detecting entries and map elements. The two-stage detection approach also proved highly effective. Once entries are detected on a map, their internal separation into number and text is very reliable; qualitative evaluation shows nearly perfect splits.

The entire pipeline is being implemented in a Python-based application called the Dialectal Map Transcriptor. Once fully integrated, the system will output transliterated textual data aligned with each map location point, ready for use in digital linguistic atlas databases.

PERSONALIZED LEARNING THROUGH COGNITIVE USER MODELING

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Cognitive characteristics play a foundational role in the development of student user models for adaptive learning systems. These characteristics encompass the mental capacities and processes that influence how students acquire, retain, and apply knowledge. Key cognitive dimensions include **knowledge level**, which covers conceptual, procedural, declarative, and episodic understanding; and **misconceptions**, which reflect persistent incorrect beliefs that require targeted instructional remediation. **Cognitive style** – such as field-dependence or reflectiveness – shapes how learners prefer to engage with information, while **cognitive load tolerance** defines the amount of information a student can manage without becoming overwhelmed. Additionally, **metacognitive skills** such as planning, monitoring, and regulation allow learners to self-direct and optimize their study behaviors.

Other vital elements include **problem-solving ability**, which captures a student's capacity for reasoning and inference across domains; and **learning goals and strategies**, distinguishing between mastery- or performance-oriented behaviors and deep versus surface approaches. Finally, **learning progression** models how understanding evolves over time, often tracked through methods such as Bayesian Knowledge Tracing, Learning Factor Analysis, or neural-network-based Deep Knowledge Tracing. Together, these cognitive characteristics inform adaptive learning technologies that personalize content, feedback, and instructional design, ultimately fostering more responsive, learner-centered educational systems.

By modeling cognitive traits such as knowledge level, misconceptions, and metacognitive abilities, intelligent educational systems can adapt instructional strategies, content complexity, pacing, and feedback to align with each learner's profile. These characteristics are used to personalize learning paths, deliver targeted feedback, adjust task difficulty, and support metacognitive growth. Real-time modeling techniques—ranging from Bayesian knowledge tracing to deep learning—enable systems to dynamically update user profiles based on ongoing student interactions with learning materials. Incorporating cognitive characteristics not only enhances personalization and engagement but also supports more effective learning trajectories, addressing individual strengths and challenges.

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This work investigates these essential cognitive dimensions and proposes a conceptual architecture for a student user profile based on cognitive modeling. The goal is to support the design of learner-aware educational systems capable of delivering a more responsive, adaptive, and personalized experience.

Keywords: user model, cognitive characteristics, adaptive learning, personalization

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INTERNATIONALISING PROGRAMMING LANGUAGES USING REGULAR EXPRESSIONS

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This paper presents an approach for internationalising programming languages through the use of regular expressions. By using the grouping constructs in regular expressions, we demonstrate a strategy to localise programming syntax based on natural languages while maintaining a unified backend representation.

Commonly, most popular programming languages are centered around English-language keywords. This poses difficulties to programmers from cultures that do not speak English [1]. Our proposal is to use regular expressions to identify language-specific constructs in localised source code, and transform them into a standardised, language-independent format that can be interpreted by a compiler.

Using the System.Text.RegularExpressions namespace found in the .NET framework [2], an in-house lexer was implemented to identify programming keywords via named capturing groups in regular expressions. These groups abstract syntax patterns across different natural languages, mapping localised constructs to their language-independent forms. Having defined a set of regular expression patterns that map each language-specific keyword to their corresponding semantic instruction (e. g. variable declaration, conditional branch), our lexer transforms input source code files by replacing the matches with their language-independent counterparts.

In conclusion, the presented approach unlocks the possibility to write computer programs in different natural languages while remaining interoperable through a unified intermediate form. By abstracting keyword semantics through named capturing groups, programmers can write code in their native language without sacrificing interoperability with other source code files written in another natural language.

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METHODOLOGICAL APPROACHES IN ENCRYPTION BASED ON THE LAPLACE TRANSFORM

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Encryption based on the Laplace transform represents an emerging direction in both analog and digital cryptography, providing a robust mathematical framework for masking, transforming, and securing signals and data. This paper analyzes the theoretical foundations, application mechanisms, advantages and limitations of this method with relevant examples.

Introduction

The Laplace transform, a classical tool in dynamic system analysis and signal processing, has recently found application in cryptography for both analog signals and digital data protection [2]. It exploits the ability to map temporal signals into a complex domain where algebraic manipulations can effectively conceal original content.

Theoretical Foundation

For a continuous function $f(t)$, the Laplace transform is $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$, with $s = \sigma + i\omega$ a complex variable [1].

Encryption Methodology

The workflow involves: (1) Pre-processing – normalize signal $(f(t))$;
(2) Transformation – compute $(F(s) = \mathcal{L}\{f(t)\})$;
(3) Masking – multiply by secret kernel $(K(s))$ to obtain $(C(s) = F(s)K(s))$;
(4) Transmission/Storage of $(C(s))$;
(5) Decryption – divide by $(K(s))$ and apply inverse transform. Modern substitution – permutation networks can be built directly in the Laplace domain [3].

Example

For $(f(t) = \sin(2\pi t))$ we have $(\mathcal{L}\{\sin(2\pi t)\} = \frac{2\pi}{s^2+(2\pi)^2})$. Choosing $(K(s) =$

$e^{\alpha s}$) yields ($C(s) = \frac{2\pi}{s^2+(2\pi)^2}\, e^{\alpha s}$), which is decrypted by dividing out ($e^{\alpha s}$) and applying ($\mathcal{L}^{-1}$).

Applications and Conclusions

Applications include encrypted control loops in cyber–physical systems, biomedical signal protection (EEG, ECG), and audio-stream encryption [4]. The Laplace transform extends classical cryptography to continuous-time signals and mixed-signal environments. Although numerical inversion and noise sensitivity remain challenges, its potential for real-time embedded security is considerable. Future work includes hybrid post-quantum schemes and hardware acceleration.

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SOME MODIFICATIONS OF THE DMP ALGORITHM FOR TESTING UNDIRECTED GRAPHS PLANARITY

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Planarity testing of graph is often a necessary step in drawing hierarchical structures (graphs, trees, schemes). Most planarity testing algorithms are based on the application of two methods. The first methods refers to the Kuratowski (Wegner) theorem and is based on finding an K_5 or $K_{3,3}$ subgraph (minor) in the tested graph. The second methods is based on the fundamental property of the closed Jordan curve, which states that no point inside the curve cannot be connected with any other point outside without crossing the curve.

Implementation of theorem-based testing algorithms is quite difficult because requires finding subgraphs (minors), a rather complicated operation. More like so, even for planar graphs, the theorem does not give a scheme for drawing the graph.

A notable representative of the second method is the algorithm is the algorithm of Demoucron-Malgrange-Pertuiset (DMP) [1-2]. In [2] this algorithm was named "the γ algorithm". Along with the planarity test, the DMP algorithm builds a set of faces that represent a planar partition of the graph. This patrition can be further used for automatic graph drawing.

In this article, are proposed some modifications of the DMP algorithm based on a depth-first search strategy [3], as follow:

- building the cover tree,
- testing the biconnectivity of the graph,
- building the set of fundamental cycles and selecting a cycle of maximum length,
- specifying the notion of segment (fragment, bridge),
- developing algorithms for building, updatting, and embeddind segments

Unlike the original algorithm, the building of the segments occurs only once, and only some segments are updated after each iteration. The proposed algorithm also works for nonplanar undirected graphs by finally constructing a planar subgraph and displaying the list of segments that cannot be embedded so as they generate edge intersections when drawing.

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COMPARATIVE EVALUATION OF AI-POWERED IMAGE GENERATION PLATFORMS

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This study offers an in-depth comparative analysis of current AI-powered image generation platforms. The goal is to pinpoint the unique features, operational performance, and inherent limitations of each solution. We investigated a selection of platforms, including Google Gemini, ChatGPT Plus, NightCafe, CGDream, and OpenArt.ai. Evaluation used several criteria: the quality of generated images, the speed of generation, their economic model (free vs. credit-based), and most importantly, how accurately they interpreted text prompts.

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Our findings reveal notable differences in the capabilities and performance across these tools. Google Gemini stands out for its fast generation and free access, consistently delivering good visual quality. ChatGPT Plus, through its DALL- E integration, provides images with superior aesthetic fidelity and an excellent ability to understand complex prompts, though its processing speed is relatively slower.

NightCafe stands out for its speed of generation and hybrid economic model, combining initial credits with the ability to accumulate them later. In contrast, CGDream demonstrated significant weaknesses in interpreting prompts. The platform often generates images that have no relation to the entered text. Compared to other platforms, even of lower quality, CGDream showed a reduced ability to correctly interpret simple queries. For example, for the proverb "A fi cu musca pe căciulă" (lit. "To be with a fly on a hat", meaning "to feel guilty"), while other platforms generated relevant images (hat, hat with a fly), CGDream returned an image of a girl with an umbrella, which illustrates a significant problem in understanding the context.

OpenArt.ai is a robust platform featuring an intuitive interface, diverse AI tools, and extensive customization options. It shows considerable stylistic versatility, capable of generating distinct conceptual representations. For example, when given "A face ochi dulci cuiva" (to flirt), OpenArt.ai creates stylized illustrations from text prompts, showing versatile aesthetics. Minor issues exist with visual consistency and file management; it's a credit-based service.

This comparative analysis suggests that choosing an AI image generation platform requires careful thought. It's vital to consider specific user needs, weighing factors like image quality, operational speed, cost, and prompt interpretation accuracy to achieve maximum effectiveness and relevance for diverse tasks.

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Keywords: AI Image Generation, Platform Comparison, Generative AI, Image Synthesis.

MATHEMATICAL ANALYSIS OF INFORMATION SECURITY THREATS IN ADVANCED METERING INFRASTRUCTURE

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Modern smart metering systems are a cornerstone of digital energy infrastructure, enabling real-time data acquisition, remote monitoring, and enhanced operational efficiency. The adoption of Smart Grid technologies and communication protocols such as DLMS/COSEM, MQTT, and NB-IoT significantly improves the quality and speed of electricity accounting. However, these advances introduce new cybersecurity vulnerabilities that make smart meters attractive targets for malicious actors. This paper proposes a comprehensive approach to mathematical modeling of information security threats in smart metering systems. The methodology integrates several modeling techniques: The STRIDE threat modeling framework is used to classify security threats, including spoofing, tampering, repudiation, information disclosure, denial of service, and privilege escalation. Data Flow Diagrams (DFD) are applied to visualize system components, data interactions, and trust boundaries, enabling identification of critical interfaces and insecure flows. Attack Trees are constructed to describe and assess potential attack scenarios, including Man-in-the-Middle (MitM) attacks, denial-of-service (DoS), firmware tampering, and command injection. Each attack scenario is quantitatively evaluated using risk metrics based on probability-impact models. The resulting risk value

$$R = P \times I$$

$R = P \times I$ supports prioritization of threat mitigation efforts. The modeling process identifies vulnerable system components, such as unauthenticated communication links, insecure firmware update mechanisms, and lack of command validation. The study concludes with practical recommendations for enhancing system security, including the implementation of encrypted communication protocols (e.g., TLS), secure boot mechanisms, anomaly detection systems, and compliance with relevant cybersecurity standards such as GOST R 57580, ISO/IEC 27019, and the EU's NIS2 directive. The proposed modeling framework can assist engineers, cybersecurity specialists, and utility operators in designing, auditing, and certifying secure smart metering infrastructures for critical energy systems.

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TEST GENERATION FOR CONTEST PROGRAMMING TASKS

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One of the most complex and routine problems in the preparation of programming competitions is the creation of tests for contest tasks. The test preparation for contest tasks is a particular case of software testing [1], but with some specific features. The tests should check:

- the computational complexity of the algorithm;
- the correctness of the algorithm on boundary conditions;
- the correctness of the algorithm on edge cases.

Moreover, the computational complexity of the algorithm is determined not only by time limits but also by memory constraints. A contest problem may have various solution approaches (for example, brute force, greedy algorithm, dynamic programming), and the tests should objectively evaluate all these approaches.

This article describes a tool [2] for generating tests for programming tasks based on a test specification.

The test specification is a description of the problem's input data in YAML format. The test specification contains the following information:

- the filename pattern for naming test files;
- the range of test case numbers;
- a description of the tests to generate;
- a description of the format of the tests.

The test format consists of one or more blocks of data lines. Each data line may contain one or more data elements separated by spaces. Each data element can be represented as an integer, a floating-point number, a string, or an array of one of these types. For each data element, additional constraints can be specified, such as value range, number of elements in an array, etc. Each data element can be named, which allows it to be used in other parts of the test specification.

Based on the test specification, the proposed tool allows automatic generation of tests for programming tasks.

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3D UNITY ROGUELITE HORROR SHOOTER

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This research project explores the design and development of a 3D Roguelite Horror Shooter that integrates procedural content generation, intelligent enemy AI, and resource-driven survival mechanics to create a dynamically immersive and fear-inducing player experience. The primary objective is to investigate how unpredictability and adaptive gameplay elements can heighten emotional engagement and tension, core components of the horror genre. Users are drawn to horror games for their ability to evoke intense emotional responses within a safe, controlled environment. The combination of fear, curiosity, and unpredictability creates a psychologically stimulating experience that heightens immersion and engagement. Horror games offer a unique sense of mastery through survival, enabling players to confront and overcome fear. Over the years, various techniques have been developed to evoke fear in horror games. These methods span multiple sensory and narrative dimensions, including visual elements such as darkness, apparitions, and surreal environments; narrative themes like nightmares, isolation, and paranoia; and audio design featuring suspenseful music, eerie sounds, and disembodied voices. Together, these elements work synergistically to create a deeply unsettling and immersive horror experience.

Procedural generation and adaptive AI further enhance replayability and tension, while the genre's aesthetic and narrative depth appeal to those fascinated by mystery and the unknown. Socially and psychologically, horror games foster resilience, thrill-seeking, and emotional exploration. The game features procedurally generated levels, original mechanics tailored to horror and roguelite conventions, and a self-contained in-game economy where ammunition and resources are intentionally scarce, encouraging tactical decision-making. Enemies are equipped with varied AI behaviors that react differently based on player actions, contributing to high replayability and emergent challenge.

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From a research standpoint, the project examines how procedural systems and AI can be applied not only for technical scalability and design efficiency but also for crafting psychological responses such as anxiety, urgency, and dread. Socially, the game promotes emotional resilience and cognitive adaptability under stress, while economically, it highlights the viability of scalable horror game development for indie markets by reducing the need for manual content creation. Future work will focus on expanding AI adaptability, integrating player emotion recognition, and exploring immersive applications in virtual reality. This study contributes to the growing field of emotionally intelligent game design and the interdisciplinary potential of horror as both a genre and a psychological simulation tool.

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Keywords: Game, Horror, Game Development, Ingame AI, Player Psychology

SEMANTIC TEST CASE GENERATION FROM CODE USING LLM

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With testing consuming up to 40–50% of software development time and the adoption of artificial intelligence (AI) in testing growing steadily across industries, the need for more intelligent and efficient testing strategies has become increasingly important. As reported in both academic studies and industrial surveys, traditional test scripting is often insufficient in fast-paced DevOps pipelines. AI-driven approaches offer a promising alternative, particularly through the use of natural language processing (NLP) and large language models (LLM) that can interpret requirements, source code, and expected behaviors to support test creation and maintenance. Industry analyses, such as those of AutonomIQ and Sauce Labs [1], suggest that up to 70% of manual test cases could be generated or improved using semantic analysis techniques.

This paper investigates the way LLM and NLP techniques are applied to generate test cases directly from source code or textual requirements. These models are usually trained on large corpora of code-comment pairs, unit test datasets, and test assertion structures. Once trained, they can generate executable testing methods by identifying logical entry points, common assertions,

and input/output patterns, primarily predicting 'what should be tested' and 'how'. Applied to user stories, NLP techniques extract actions, conditions, and expected results, mapping them into structured test steps. For source code, LLMs interpret method names, control structures, and API behavior to synthesize meaningful unit tests. In this study, we apply ChatUniTest [2], an open source Java-based tool, to evaluate its LLM-powered test generation workflow (generate-validate-repair) on a real system under test (SUT).

Our evaluation shows that LLM-generated tests can meaningfully complement manually written ones, covering basic logic and frequent corner cases while reducing boilerplate work. Although the current generation of tools still benefits from developer supervision, their semantic capabilities open new directions to streamline QA workflows and improve early-stage testing activities - with possible applications in education as well [3,4].

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CLASSIFICATION ELABORATION FOR THE DIGITIZED HANDWRITTEN ARCHIVE OF SCHOOL MATH PROBLEMS OF THE TEACHER AND SCIENTIST B. CINIC

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The authors of this work consider the handwritten archive of math problems prepared and collected by the mathematician Boris Cinic during his school teaching. The problems in the archive were elaborated by B. Cinic for math lessons in the senior classes of the school during the period 1990–2013. All the problems of the archive are written by the author on the punch cards as individual tasks for his students.

[1] *Speaking author*: T. Verlan

The authors of this work are currently digitizing this archive to prepare a problem book. A part of the problems passed this process already. The information from 240 processed punch cards has been recognized, and the LaTeX code generated by MathPix and GPT online services has been manually verified and corrected [1]. All the processed problems, however, need some regularization or classification. Classification of math problems depends on different criteria, such as field of mathematics, complexity level, solution method, structure, etc. The content, presentation form, and many other criteria are also important.

1. At the initial step of the criteria selection, we asked ChatGPT for help. As the starting point, we asked for an abstract classification of math problems in general. For example, ChatGPT proposed to us the following criteria for the content of the problems: ***Arithmetical; Algebraic; Geometrical; Combinatorial; Theory of numbers; Logical; Probabilistic and statistical***.

2. However, since we deal with the problems for school students, we should select the criteria according to the school level. ChatGPT proposed to us the following criteria: ***By the nature of the mental activity; By didactic goals; By level of difficulty; and By form of presentation***.

3. As applied to our specific problem set, we started from the proposed criteria but oriented on the existing content. To make a decision, we had to analyze each problem or the group of problems (as B.Cinic intended), select proper criteria from those proposed by ChatGPT, or choose our own, based on our experience, intuition, knowledge, and existing math textbooks: ***Free work; Arithmetic; Algebraic; Geometric (with and without a Picture); Trigonometric; Logic problems; Problems with a suggested Solution hint***. Sometimes, these criteria can be applied in combination, e.g., ***Free work on Algebra; Geometric problem with a Solution hint and a Picture.***

Work on the selection of criteria, their formulation, and clarification, as well as on the distribution of tasks or sets of tasks according to criteria continues.

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ASPECTS OF AI MODELING THE PERCEPTION OF CREDIBILITY OF INFORMATION ON THE WEB

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The abundance of information emerging from different Web sources is an enormous challenge for us, active readers. We are forced to make an effort to select truly valuable - credible information. Beyond the multitude of existing media sources that create content, there are platforms dedicated to promoting it.

Platforms such as Google, YouTube, Facebook and TikTok use algorithmic classification and recommendation mechanisms that not only influence what users see, but also shape how they perceive the credibility of information [1,2]. Therefore, it is important to take into account the role of AI and algorithms in shaping perceived credibility, highlighting the complex interaction between system-driven content delivery and cognitive biases at the user level.

Even if the possibility of personalizing the display of content seems useful, this imposes an emotional or popular character - but potentially with low credibility. Complex multidisciplinary analysis combining media studies, cognitive psychology, and computer science leads us to the idea that search result positioning, content repetition, source familiarity, and interface design influence users' judgments of credibility. The systems are programmed on the psychological heuristics that users use - such as assuming that top-ranked results are more trustworthy or equating popularity with accuracy.

Case studies examined include the spread of low-credibility medical information during global crises, the behavior of recommender systems in multilingual contexts, and trust biases in educational and youth settings. The findings suggest that algorithmic systems do more than simply reflect users' behavior - they actively shape perceptions of truth, often without users being aware of it.

An important aspect is the discussion of the ethical implications of the design of such algorithms and the need for greater transparency in artificial intelligence systems. Accordingly, it is necessary to develop credibility-aware algorithms and highlight the importance of integrating digital literacy education to help users critically evaluate online information.

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MODEL-DRIVEN ENGINEERING IN COMPUTER BASED INSTRUCTION COURSES DEVELOPMENT

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Model-Driven Engineering (MDE) is a software engineering approach that uses abstract models and automated transformations to drive developmen. In computer-based instruction (CBI) course development, MDE involves defining platform-independent course models that can be automatically transformed into courseware for multiple e-learning platforms. This approach automates and structures course creation, yielding improved r eusability, scalability, and maintainability of instructional systems [1].

Model-Driven Engineering (MDE) offers significant advantages in developing computer-based instruction (CBI) courses. Firstly, MDE facilitates a high level of abstraction, enabling educators and instructional designers to focus on the pedagogical structure and learning objectives rather than technical implementation details.

Secondly, by employing automated model transformations, MDE substantially reduces development time and minimizes errors, enhancing productivity and consistency across multiple instructional materials. Furthermore, MDE inherently supports reusability; instructional components and course structures modeled once can be adapted and reused efficiently across various educational contexts and platforms. This capability not only optimizes resource allocation but also ensures greater scalability of instructional programs.

Additionally, the explicit representation of instructional designs in models promotes better collaboration among multidisciplinary teams, enabling clear communication between pedagogical experts and software developers. Consequently, MDE contributes to improved maintainability and easier updates of CBI systems, allowing instructional content to remain aligned with evolving educational standards and technological advancements.

The main components of the learning process model are stereotypical learning situations: Information, Question, Check, Test, Branching, etc.[2]

In this model, instructional knowledge is organized as frames, each describing a stereotypical instructional situation with specific semantic attributes a nd relationships; causal scenarios shape the dynamics of the instructional process, defining how transitions between instructional situations are governed by the actions and responses of the learner; adaptability is built into the course structure, allowing for customization of the learning path based on the individual performance and needs of the users

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PREDICTIVE ANALYSIS OF UNIVERSITY ENROLLMENT TRENDS USING STATISTICAL MODELING AND AI TECHNIQUES

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This project presents a data-driven approach for analyzing and predicting trends in university student enrollment, based on historical data from national birth statistics and institutional records from 2005 to 2023 [1]. The analysis includes the calculation of key statistical indicators mean, median, mode, annual growth rate, and year-over-year variation for various academic specialities. To forecast future enrollment, the project integrates several predictive models, including linear regression, polynomial regression (degrees 2 and 3), exponential trends, and autoregressive models. These models are applied per speciality, allowing tailored forecasts with associated confidence intervals. The correlation with national natality data enables a broader socio-demographic context for interpretation. The application was implemented using Python and Streamlit [2], offering an interactive interface where users can explore current trends and generate forecasts for up to 10 years into the future. Visualization tools include dynamic graphs and tables, supporting strategic decision-making in educational planning and resource allocation. The project highlights the potential

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of combining statistical methods with machine learning techniques to support data-informed educational policy and institutional development.

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PUBLIC AWARENESS AND PERCEPTIONS OF SMART MOBILITY: A SURVEY-BASED ANALYSIS

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The "Smart City" concept has gained global traction as a paradigm for urban innovation and sustainable development, leveraging ICTs to enhance quality of life and resource management. However, the absence of a universal definition highlights its conceptual complexity, with truly effective implementations that require measurable outcomes in resource efficiency, economic innovation, and inclusive governance. Smart mobility emerges as a critical pillar of this transformation, integrating solutions such as ITS, autonomous vehicles, and multimodal systems to address congestion, safety, and environmental impact [1]. Although European initiatives demonstrate success in areas like electric mobility and MaaS platforms, persistent challenges include generational adoption gaps and insufficient attention to social dimensions such as accessibility and urban quality of life [2].

Our research is based on a survey of Romanian public perceptions that reveals a significant awareness of smart mobility concepts but limited familiarity with local implementations, creating a trust gap exacerbated by cost barriers, interoperability issues, and poorly understood user needs. Citizens prioritize convenience and affordability in transport choices over environmental considerations, despite expressing interest in sustainable alternatives like micro-mobility. Willingness to adopt smart solutions in the medium to long term is tempered by inadequate policy frameworks, insufficient incentives, and unresolved data privacy concerns [3]. The findings underscore the need for coherent policies that bridge communication gaps between stakeholders, coupled with infrastructure

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development that balances practical user needs with sustainability objectives, ensuring that smart mobility delivers on its transformative potential.

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SECURE EXECUTION OF PARTICIPANTS' CODE IN AUTOMATED EVALUATION SYSTEMS

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Automated programming contest systems (online judges) compile and run untrusted code submitted by participants, evaluating its correctness under strict time and memory limits. Ensuring the secure execution of these solutions is crucial: without proper isolation, a malicious submission could access secret test data, crash the judging system, or gain an unfair advantage. This paper examines the motivations and challenges of securely executing contest submissions and outlines the strategies and constraints involved in designing a robust code sandbox environment.

The system must prevent forbidden actions (such as file access beyond the allowed scope, network communication, or process tampering) while simultaneously enforcing resource limits to detect inefficiencies or abuse (e.g., infinite loops or memory hogs) [1, 2]. These safeguards should impose minimal performance overhead, since contest judges often run thousands of submissions under tight time constraints. Another challenge is supporting a variety of programming languages and run-time behaviors—some programs may spawn multiple threads or processes—within a unified sandbox framework.

We highlight both architectural and platform-level constraints that shape sandbox design. At the system architecture level, the judging environment

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must isolate untrusted execution from the critical infrastructure. Such design decisions are influenced by scalability requirements and the need to support multiple operating platforms. At the operating system level, sandboxing capabilities vary: the security primitives available on Linux versus Windows fundamentally influence how a secure execution environment can be implemented on each.

In the Linux OS sandboxing relies on kernel-level mechanisms such as `chroot` for file isolation, user privilege drop, and syscall filtering via `ptrace` or `seccomp`. Modern approaches combine namespaces and `cgroups` to create lightweight containers with enforced CPU and memory limits.

In Windows OS, due to the absence of native equivalents for `ptrace` and namespaces, isolation typically involves restricted user accounts, Job Objects, and virtualization tools such as Windows Sandbox or Hyper-V containers, offering stronger security at the cost of higher resource overhead.

In summary, secure code isolation is critical to fairness and reliability in automated contests. Sandboxing ensures integrity by preventing interference, resource abuse, or unauthorized access, enabling scalable and trustworthy evaluation across all submissions.

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BEHIND THE WORDS: WHAT DRIVES TEXT COMPLEXITY?

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The complexity of texts remains a central concern in linguistic research, education, and Natural Language Processing (NLP). Traditional readability formulas, such as Flesch-Kincaid and Gunning Fog, offer a superficial assessment of text difficulty, often failing to capture deeper linguistic structures. This study proposes a feature-based analytical approach to understanding text complexity, focusing on specific linguistic attributes—lexical diversity, syntactic complexity, named entity recognition (NER), text coherence, and readability—to determine

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their relative impact on complexity and their stability across different subject domains.

The research is guided by two core objectives: (1) to rank the specific linguistic features that exert the most significant influence on text complexity, and (2) to assess the consistency of these effects across diverse text domains. Recent advancements in NLP and the rise of large language models (LLMs) provide the foundation for this study, enabling a more sophisticated analysis of textual semantics and structural properties. These technological developments allow for a refined understanding of complexity beyond traditional readability measures.

Methodologically, the study employs a two-phase empirical analysis. In the first phase, domain-specific texts are examined to identify dominant complexity features. In the second phase, a cross-domain comparison is conducted to assess the generalizability of these features. The findings indicate that while most linguistic attributes exhibit stable influence across domains, lexical diversity and NER show variability depending on thematic context. These results underscore the importance of considering domain-specific interpretations when evaluating text complexity.

The study concludes that a feature-level approach provides a more nuanced and comprehensive understanding of text complexity than traditional models. Certain linguistic features serve as robust complexity indicators across domains, while others require contextual adaptation. These insights have significant implications for adaptive educational systems, content recommendation algorithms, and AI-driven applications designed to tailor textual materials to reader proficiency levels. By refining methodologies for assessing complexity, this research contributes to the ongoing discourse on automated text evaluation and its applicability in various fields.

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BRINGING WEB CONTENT TO LIFE - A COMPARATIVE STUDY OF TTS TECHNOLOGIES

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In the age of voice interfaces and AI-driven interaction, Text-to-Speech (TTS) technologies [1] play a key role in enhancing accessibility, personalization, and interactivity across digital platforms. This brief study compares two widely used Python-based TTS tools: gTTS (Google Text-to-Speech) [2] and Google Cloud Text-to-Speech, focusing on their integration into web projects and user benefits [3].

User Benefits TTS technology benefits users in multiple ways:

- Accessibility: Enhances usability for visually impaired users or those with reading difficulties.
- Language Learning: Supports pronunciation practice and comprehension through native voice output.
- Content Engagement: Audio-enriched websites or e-learning platforms increase user interaction and retention.
- Hands-Free Interaction: Supports voice-enabled browsing or listening on the go. By choosing an appropriate TTS engine, developers can match the needs of their target audience while maintaining development efficiency. **Conclusion** While gTTS is perfect for quick integration and experimentation, developers seeking high-quality voice output and control should consider Google Cloud TTS. Both tools demonstrate the power of AI in transforming static web content into dynamic, inclusive, and engaging experiences.

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EXTENDED MODEL OF SOS REGULON SWITCHING: THE ROLE OF COOPERATIVE PROCESSES IN SYSTEM REGULATION

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Cooperative processes are fundamental to the self-organization of biological systems, enabling rapid and coordinated responses to environmental challenges. This paper proposes an extended computational model of the bacterial SOS regulon switching under stress conditions induced by exogenous mutagenic factors, such as DNA-damaging agents. The model focuses on cooperative interactions, including autoregulation of the LexA repressor, activation of the RecA coprotease, cleavage of the repressor, positive feedback of RecA, DNA repair, and re-entry into the repressed state. These processes are critical for bacteria to efficiently respond to genotoxic stress and restore quiescence after DNA repair.

Unlike previous models, this work incorporates detailed cooperative binding dynamics and feedback mechanisms, with parameters derived from experimental scientific literature, to better capture the non-linear behavior of the SOS system.

The model is formalized using a system of ordinary differential equations, simplified using Tikhonov's theorem—a mathematical approach to reduce complexity in systems with multiple timescales—based on the hierarchy of characteristic times and molecular concentrations. Simulations, implemented using Petri nets alongside the differential equation framework, provide an animated representation of the system's evolution, from the healthy state (absence of mutagenic stress) through the cellular response to DNA damage, repair of lesions, and return to the initial quiescent state.

The results, visualized graphically, depict the activity of operators and promoters, the dynamics of *lexA* and *recA* gene expression, the concentration profiles of LexA, RecA, and activated RecA proteins, and the formation and dissociation of filaments. These findings align with experimental data, validating the model's predictive power.

Such models enhance our understanding of trigger-like systems, which can be successfully applied in bioinformatics domains, including molecular computing, membrane computing, and DNA-based computing. Furthermore, the model highlights the role of biological switches and bistable systems, which exhibit robust, massively parallel computational properties. These characteristics make them promising for addressing complex computational challenges, such as NP-complete problems, by leveraging the inherent parallelism and scalability of molecular processes.

RELIABILITY OF SERIAL-PARALLEL AND PARALLEL-SERIAL NETWORKS WITH A VARIABLE NUMBER OF SUBNETWORKS AND A CONSTANT NUMBER OF UNITS

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Reliability analysis in networks of series-parallel and parallel-series configurations plays an important role in the study of technical systems. Often, such networks are included as component parts in complex architectures, and their overall performance is directly influenced by the reliable behavior of the substructures that compose them. Most studies in the field are based on static models, which assume the constancy in time of the operating probabilities of the component elements, as well as a fixed number of subnetworks and units. Since in modern communication systems, the operating duration of nodes and connections can vary significantly and is often represented by discrete random variables, it is relevant to study different dynamic probabilistic models and their reliability. In this study, we identified and described two new dynamic probabilistic models for reliable networks of the Serial-Parallel and Parallel-Serial type with a constant number of units in each subnet and a variable number of subnets. These models are characterized by the fact that the number of units in each subnet is constant, the network units are independent random variables identically distributed with geometrically distributed lifetimes, and the number of subnets follows the Poisson distribution. Based on these distributions, formulas were derived for evaluating the reliability of the networks. For the resulting models, we performed the estimation of statistical characteristics (mean, dispersion) and the estimation of parameters by the maximum likelihood method.

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STUDY ON THE LONGEVITY PERSPECTIVES OF INFORMATION TOOLS USED IN MEDICAL DIAGNOSTICS

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The overall aim of this paper is to study the subject of the longevity of the existing informational tools (such as knowledge-based systems, decision support systems, knowledge and data warehouses, and scoring systems), related to medical diagnostics, based on the opinions of the medical practitioners gained through a survey. It will allow us, in the first stage, to identify the constraints and limitations of these tools. In the second stage, we will be able to propose solutions in order to minimize the negative effects of the most important ones, detected in the first stage.

In the medical diagnostics domain, there is no single approach sufficient. The widespread use of knowledge-based, data-driven and mixed approaches in the decision-making process is determined by how they support decisions under uncertainty and complexity. The knowledge-based approach relies on expertise, rules, heuristics, and past experiences stored in knowledge bases. It provides explainable decisions, facilitating consistency and standardization in repetitive decisions [1]. The data-driven approach, in its turn, relies on patterns, correlations, and trends. It enables real-time, adaptive decision-making from big data (often via statistical analysis or machine learning) and can identify hidden patterns beyond human intuition.

Scoring systems play an important role in both knowledge-based and data-driven approaches (particularly in diagnostics, prognosis and triage), with the main contribution in providing a structured and quantitative way of evaluating and comparing different options or alternatives. The scoring systems allow the prioritization of factors according to their relative importance, increase the level of objectivity, and ensure consistency in decision-making. These facts have led to the widespread use of this tool to assist the decision-making process.

This study seeks to gain a better understanding of changes over time in the use of informational tools depending on the approach used in their creation. As an outcome, solutions aimed at mitigating the disadvantages associated with the use (both explicitly and implicitly) of traditional scoring systems were proposed.

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of traditional scoring systems" and the institutional project 011301 "Information systems based on Artificial Intelligence" (Republic of Moldova) have supported this research.

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IMPROVING THE INTERPRETABILITY OF SCORING SYSTEMS THROUGH GRAPHICAL VISUALIZATION

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The main importance of scores in the decision-making process is to provide a structured and quantitative way of evaluating and comparing different systems or options or alternatives. This can lead to more data-based and evidence-based decisions. These systems allow prioritizing factors according to their relative importance, increasing the level of objectivity, promoting consistency and more transparency in decision-making. The aforementioned facts caused, at the first stage, a wide spread of this important decision-making support tool. However, real use has also highlighted some important limitations. These restrictions can cause hesitations in the use of score systems.

The purpose of this paper is to analyze the use of an alternative interpretation of the score system, and minimize the constraints and limitations of traditional score systems. As a field of implementation and validation of the proposed approach, the development of decision support systems in medical diagnosis was chosen, namely - the development of the Doppler ultrasound score in cirrhotic portal hypertension. Structurally, the score corresponds to the well-known score and systems. This involves: the selection of a system of parameters, which describe the problem area; determining the weight of the parameter in part in the final score; interpretation of the final score. Both the score itself and its interpretation are represented in numerical - tabular form [1]. A deeper analysis showed a possible emergence of reluctance regarding the level of confidence

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in the interpretation of the score. Considering that there is also a graphical representation of the information, it was decided to develop and test a new version of score, which also offers an alternative graphical version of the interpretation – namely in the form of a "spider" or "radar" diagram.

The validation process that followed previously showed that offering both forms of representation of the interpretation of a scoring system (numerical-tabular and graphic-diagram "spider") allows for misunderstanding and a degree of distrust in the conclusions generated by the score. The proposed approach, methods, and algorithms provide a methodology that can be used to modify existing scores or to develop new systems.

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CULTIVATING AWARENESS: USING AUGMENTED REALITY TO COMBAT PLANT BLINDNESS IN HOME ENVIRONMENTS

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Plants are common in daily spaces, but are often overlooked. Although vital for ecosystems and life, they receive little attention. Recent advances in immersive technologies, particularly Augmented Reality (AR) [?], open up novel ways to bridge this perceptual gap. AR can render the invisible visible, translating silent biological signals into interactive visual experiences tailored to everyday users. What if the houseplant next to your window was silently signaling distress - and you didn't even know its name? Each day, countless houseplants suffer not from overt neglect, but from a more insidious phenomenon known as plant blindness: the cognitive bias that causes individuals to overlook, undervalue, or misidentify the plants in their immediate environment. This perceptual gap results in decreased awareness of plant care needs, ecological functions, and

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the intrinsic value of botanical life. Rooted in educational, cultural, and psychological factors, plant blindness undermines both environmental literacy and sustainable behaviors within domestic spaces. Given the increasing integration of technology into daily life, this research argues that Augmented Reality (AR) offers a promising solution. By making plant features, identities, and needs visible through interactive, real-time digital overlays, AR has the potential to counteract plant blindness, fostering a more informed and empathetic relationship between humans and their home flora. Furthermore, by incorporating elements of gamification, such as progress tracking, visual rewards, plant care challenges, and interactive storytelling, AR applications can appeal not only to adults, but also to younger audiences. For example, children can be encouraged to 'level up' by watering their plants on schedule, identifying leaf structures, or learning fun facts about photosynthesis through playful AR experiences. This approach turns passive observation into active learning and cultivates early environmental awareness. In particular, it provides an engaging and educational experience for children, helping them develop empathy, responsibility, and curiosity about the natural world from a young age. Thus, integrating AR with gamified plant interaction offers a dual benefit: improving plant care at home while fostering cross-generational ecological literacy and stewardship.

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MANAGING PDF TEXT LAYERS FOR VISUALISING AND COMPARING TRANSLITERATED TEXT

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This paper explores the implementation of a visual tool made to compare and search for text that has been optically recognised. This is done as to both be able to search through and render the different text layers inside a given PDF, after creating them, with the scope of creating a tool to help in visualising and highlighting the differences from Romanian text written in Cyrillic and Latin scripts in the given PDF, based on already implemented Cyrillic optical character recognition (OCR).

This program uses the HTML formatting for OCR text (hOCR) meant to not only store text data, but also store font size and word coordinates for easy processing after the character recognition step. This interface uses the PySide [1] front-end package, used to visualise two documents: the original PDF and the transliterated PDF, along with a search function to highlight the same word in both views regardless of their scripts.

This implementation aims to create a basis for a user-friendly program intended to assist archivists and digitisation specialists in curating historical documents, while also supporting general users interested in exploring and learning from these documents. Future versions of the project will expand on this work by incorporating editing features to further enhance usability.

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A WSI-BASED ASSESSMENT OF HTTP SECURITY HEADERS IMPLEMENTATION ON POPULAR WEBSITES IN THE REPUBLIC OF MOLDOVA

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Securing web communications is a fundamental pillar of cybersecurity. According to the https://owasp.org/Top10/, at least five of the most common vulnerabilities can be prevented through the proper use of HTTP security headers. OWASP recommends integrating these headers into a "defense-in-depth" strategy, emphasizing the need for their rigorous and consistent implementation across all web applications. As such, the systematic monitoring and evaluation of these headers becomes not only timely but also critical [1-2]. To analyze the implementation of HTTP security headers, the Web Security Index (WSI) has been proposed - a composite framework that quantifies the degree of implementation across three main dimensions: Content Security, protecting content through policies such as HSTS, SRI, Referrer-Policy, and proper cookie configuration (Secure, HttpOnly, SameSite); Cross-Location Security, controlling interactions with external sources through CSP, Access-Control-Allow-Origin,

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CORP, COEP, and COOP; Network and Protocol Security, targeting secure transport infrastructure by adopting modern technologies like HTTP/2, validating SSL/TLS certificates, and implementing DNSSEC.

The study's methodology included the analysis of the top 10k websites accessed from the Republic of Moldova, selected using data from the CrUX[3] dataset, which reflects real Chrome user experience rather than just DNS traffic and provides monthly updates with geographic granularity. The results obtained for the Republic of Moldova indicate an average WSI score of 49.12 out of 150, with the following component distribution: Content Security: 15.59 - deficiencies in HSTS, SRI, and cookie configuration; Cross-Location Security: 3.67 - weak implementation of CSP, CORP, and COOP; Network and Protocol Security: 29.85 - partial SSL certification and limited DNSSEC implementation.

The study highlights that, although there are ongoing regulatory and digitalization efforts, the practical implementation of security headers remains uneven. The WSI provides a framework for monitoring and could be incorporated into cybersecurity risk assessment strategies, thereby enhancing users' digital trust. Such a holistic approach not only facilitates continuous monitoring and prompt response to threats but also aids in identifying trade-offs between security and performance. Given the importance of response times and user experience, adopting these methodologies becomes essential to ensure that security measures do not negatively impact daily operations.

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GENERATIVE AI AND WORKFORCE: A SYNERGISTIC FUTURE WITH LLMS

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The advent of **Generative AI** – particularly Large Language Models (LLMs) and Transformer Architecture (*Vaswani et. all, 2017; Sanseviero et. all, 2025*) – signals a paradigmatic transformation in labor ecosystems, compelling a fundamental reimagining of workforce architectures. This paper advances three core

theses: (1) LLMs will radically restructure labor markets through dual mechanisms of *task automation* (including high-complexity cognitive and creative functions) and *capability augmentation*, (2) this disruption necessitates viewing human-AI interaction as a *collaborative continuum rather than a competitive hierarchy*, and (3) the trajectory of this transformation remains contingent on *proactive institutional and individual adaptation*.

Central to this analysis is LLMs' emergent proficiency in natural language mastery, contextual reasoning, and cross-domain knowledge synthesis — capabilities that redefine the boundaries of machine collaboration. We contextualize these technological developments within intersecting economic pressures (productivity demands), societal expectations (the meaning of work), and regulatory challenges (AI governance frameworks).

We propose an action framework for stakeholder groups:

- **Individuals** must cultivate meta-skills (critical judgment, complex problem-solving) and embrace lifelong adaptive learning;
- **Organizations** must adopt augmentation-by-design – structuring workflows so AI amplifies human expertise (e.g., LLMs as research assistants for scientists) while upskilling teams to leverage AI as co-intelligence;
- **Policy/Education Systems** need anticipatory governance models, including modular credentialing systems and AI-adjusted labor policies.

We contend that the future of work isn't predetermined by technology. Instead, it's a co-constructed state requiring deliberate human stewardship to create an equitable, human-centered paradigm where AI amplifies, rather than diminishes, workforce potential.

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SECTION 5: DIDACTICS OF MATHEMATICS AND INFORMATICS

ALGORITMUL APLICĂRII METODEI DESCOPERIRII ÎN PREDAREA-ÎNVĂŢAREA MATEMATICII ÎN LICEU

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Metoda descoperirii e una dintre cele mai eficiente metode de formare a competenţelor specifice matematicii în liceu. Considerăm că aplicarea metodei descoperirii în procesul educaţional ar trebui să fie axată pe respectarea următorului algoritm:

- Selectarea problemei/sarcinei/studiului de caz supuse/supus cercetării;
- Determinarea strategiei de abordare a problemei investigaţionale;
- Elaborarea planului de aplicare a metodei descoperirii;
- Colaborarea didactică privind definitivarea managementului aplicării metodei descoperirii în cadrul lecţiei de matematică;
- Elaborarea proiectului didactic al lecţiei;
- Realizarea planului elaborat: a) prezentarea problemei; b) realizarea activităţii investigaţionale; c) evidenţierea produsului matematic descoperit (teorema, formula, proprietatea etc.);
- Aplicarea noului produs matematic în diverse contexte;
- Formularea concluziilor şi a recomandărilor rezultate din activitatea realizată în cadrul lecţiei.

Menţionăm că problema cercetată în cadrul aplicării metodei descoperirii în procesul predării-învăţării matematicii este întotdeauna derivată dintr-un context real; ea porneşte mereu de la ceva ce face parte din mediul înconjurător al elevilor. Sarcina trebuie aleasă cu grijă, pentru a putea fi rezolvată de toţi elevii, inclusiv de cei cu performanţe mai slabe. Sarcina trebuie selectată astfel încât să faciliteze soluţii diverse şi o formă de generalizare.

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ÎNVAŢĂ PROGRAMAREA, VERIFICĂ CU CODERUNNER PE MOODLE

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În contextul digitalizării accelerate a educaţiei, integrarea instrumentelor interactive în procesul instructiv-educativ devine crucială pentru creşterea eficienţei actului educaţional. CodeRunner pe Moodle se conturează ca un instrument indispensabil, care asigură atât monitorizarea, cât şi aprecierea obiectivă a progresului elevilor. De la o funcţie strict evaluativă [1], acesta a evoluat într-un instrument interactiv de învăţare, aplicabil atât în cadrul învăţământului la distanţă, cât şi în cel tradiţional, care captează atenţia cursanţilor prin conţinut multimedia dinamic [2], acoperind toate etapele procesului educaţional - de la formare iniţială până la evaluare finală. Utilizarea plugin-ului CodeRunner asigură o motivaţie crescută şi o atitudine pozitivă în rândul elevilor printr-o experienţă de învăţare personalizată şi interactivă, contribuind în acelaşi timp la eficientizarea timpului profesorului şi la optimizarea efortului didactic.

CodeRunner permite crearea de itemi unici parametrizaţi şi individuali pentru fiecare utilizator, cu evaluarea automată, cu feedback instant şi obiectiv, contribuind la dezvoltarea unei autoevaluări constructive şi continue cu consolidarea şi aprofundarea cunoştinţelor; este ideal pentru predarea limbajelor de programare şi reduce semnificativ riscul de copiere, stimulează gândirea critică şi autonomia în învăţare. Un test creat cu ajutorul CodeRunner pe Moodle respectă criterii esenţiale de calitate, precum Aplicabilitate - este uşor de administrat şi interpretat; Fidelitate - produce rezultate constante; Obiectivitate - evaluarea este uniformă şi lipsită de subiectivism; Validitate - măsoară exact ceea ce îşi propune, acoperind în mod echilibrat conţinuturile predate [1]. Implementarea acestor modele de itemi a fost testată cu succes în cadrul Proiectului de inovare şi transfer tehnologic "Creşterea performanţei academice la disciplina Informatică/TIC din contul implementării unor instrumente standardizate de evaluare pe platforma de eLearning Moodle", aprobat prin ordinul MEC nr. 364 din 18.03.2025.

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DESPRE FUNCŢII MULTIVALORICE

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Sunt analizate anumite funcţii multivalorice atât în domeniul real, cât şi în domeniul complex, cum ar fi:

1. Funcţiile radicale de ordinul $(2n)$ ia două valori în domeniu real, iar în mulţimea numerelor complexe ia $(2n)$ valori.

2. Funcţia exponenţială, fiind monotonă, poseda inversa sa - funcţia logaritmică, care la rândul său este multivalorică în domeniul complex.

3. Funcţiile inverse pentru funcţiile trigonometrice şi hiperbolice.

În lucrare sunt examinate mai multe exemple nestandarte, cum ar fi:

Funcţia neperiodică $f(x) = (n+1)x - n(n+1)$, $n \in Z$, $n \leq x \leq n+1$ cu funcţia inversă multivalentă:

1. $n = 0, y = x, 0 \leq x < 1, y(0) = 0, y(1) = 1.$
2. $n = 1, y = 2x - 2, 1 \leq x < 2, y(1) = 0, y(2) = 2.$
3. $n = 2, y = 3x - 6, 2 \leq x < 3, y(2) = 0, y(3) = 3.$
4. $n = 3, y = 4x - 12, 3 \leq x < 4, y(3) = 0, y(4) = 4.$
5. $n = -1, y = 0, -1 \leq x < 0.$
6. $n = -2, y = -x - 2, -2 \leq x < -1, y(-2) = 0, y(-1) = -1.$
7. $n = -3, y = -2x - 6, -3 \leq x < -2, y(-3) = 0, y(-2) = -2.$

$f^{-1}(0) = [-1; 0] \bigcup Z$ - o mulţime de puterea continuumului.

$f^{-1}(-1)(y)$, pentru $y \neq 0$ este o mulţime numărabilă.

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CLASSIC NUMBER PROBLEMS SOLVED BY ALGORITHMS WITHOUT CONTROL STRUCTURES: EDUCATIONAL INSIGHTS

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This study addresses several classical problems involving natural numbers, solved through algorithms explicitly designed without the use of control structures such as loops, conditionals, or selectors. The approach relies on mathematical concepts, mainly number theory and properties of logarithms, to formulate direct computational methods.

We present solutions to problems including calculating the number of digits of a natural number, extracting leading and trailing digits, reversing digits, and identifying the larger or smaller of two integers. Additionally, we explore digit-based computations such as summing digits, finding the maximum digit, checking divisibility, detecting the presence of specific digits, extracting digits at given positions, and calculating digit-related properties - all without loops or conditionals.

These problems highlight how fundamental mathematical reasoning can guide algorithmic design in a straightforward and elegant manner. By eliminating control structures, the approach encourages a deeper understanding of the underlying mathematics rather than focusing solely on programming syntax.

The algorithms have been implemented in C++ and JavaScript, demonstrating practical applicability in contemporary programming contexts. Furthermore, the role of mathematics in optimizing algorithms is emphasized, aiming to convince students of its essential importance. Often, students believe they can become proficient programmers without a solid mathematical foundation; this work seeks to challenge that hypothesis by illustrating how mathematics enriches algorithmic thinking and problem solving.

POTENŢIALUL JOCURILOR TEMATICE PE MOODLE PENTRU CONSOLIDAREA CUNOŞTINŢELOR

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Definiţiile, conceptele, terminologia tematică - toate acestea şi altele pot fi consolidate în mod eficient prin jocuri educaţionale de tip ***Crossword***, ***Cryptex***, ***Hangmand***, desfăşurate pe platforma de eLearning Moodle, generate aleatoriu la fiecare lansare fie din **glosare tematice**, fie din **itemi cu răspuns scurt**. Asemenea jocuri permit respondenţilor să-şi consolideze cunoştinţele tematice într-un mod **antrenant şi interactiv**; pot fi utilizate inclusiv la evaluarea intermediară şi/sau finală prin selecţia aleatorie a articolelor **din toate glosarele / toate colecţiile aferente**, asigurând acoperirea a 100% din materia verificată. Pe de altă parte, în afară de funcţia de bază ca bancă comună de termeni cheie şi generarea jocurilor variative, glosarele au şi alte utilizări, ca *zonă comună pentru lucrul interactiv al studenţilor de adăugare-comentare-evaluare reciprocă a definiţiilor, cunoaşterea reciprocă şi împărtăşirea celor mai bune practici, stocarea datelor personale pentru cunoaşterea reciprocă, a imaginilor şi fişierelor multimedia, videoclipuri etc.*

Itemii cu răspuns scurt şi/sau Glosarele terminologice pot fi uşor pregătite în medii cunoscute de marea majoritate a profesorilor, precum MS Word, MS Excel, cu verificarea-corectarea eficientă a erorilor de sintaxă, după care pot fi convertite şi încărcate automat pe Moodle.

Cunoaşterea caracteristicilor unor obiecte, produse etc. poate fi uşor verificată prin itemi de **potrivire a unor răspunsuri scurte generate aleatoriu** din colecţii tematice. Acest mod de generare necesită gruparea atentă în colecţii a itemilor cu răspunsuri cât mai omogene: de exemplu cu *răspuns numeric*, *de tip dată*, *de tip cuvinte* etc. astfel, încât să nu fie posibil de extras sau determinat univoc răspunsul din enunţ fără a avea cunoştinţele necesare.

Itemi cu răspuns singular vis-a-vis de jocul Milionar. Itemii cu răspuns singular sunt statici şi răspunsurile pot fi destul de rapid colectate, îndeosebi prin forţă brută sau când numărul celor evaluaţi este mare. Jocul Milionar generează la fiecare lansare o succesiune individuală de itemi, mult mai greu de colectat şi de identificat/regăsit răspunsurile în fişuici/copiuţe dat fiind numărul extraordinar de mare al variantelor posibile. Numărul de variante diferite se calculează după formula:

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$$C(n,k) = n!/(k!(n-k)!)$$

De exemplu, pentru un joc generat în baza a 100 de obiecte (itemi/cuvinte) cu alegerea aleatorie a 15 obiecte, numărul total de variante diferite este de circa $2,5 \cdot 10^{17}$. Pentru un item de potrivire a unor răspunsuri scurte aleatorii generat din 10 obiecte luate câte 4 numărul de variante posibile este = 210. Aceste numere sunt suficient de mari pentru (1) a asigura exemplare diferite într-o evaluare sincronă a mii de respondenţi şi (2) a preveni fraudarea prin fişuici pregătite în prealabil.

Instrumentele amintite mai sus sunt aplicabile pentru cea mai mare parte a materiilor studiate de elevi, studenţi; sugerând că este mai convenabil de învăţat decât de copiat. Experimentarea acestor instrumente a fost realizată pe platformele de eLearning ale USM `https://moodle.usm.md` şi CEITI `https://moodle1.ceiti.md/login/index.php/` în cadrul Proiectului de inovare şi transfer tehnologic "*Creşterea performanţei academice la disciplina Informatica/TIC din contul implementării unor instrumente standardizate de evaluare pe platforma de eLearning Moodle*", aprobat prin ordinul MEC *nr.364 din 18.03.2025.*

DESPRE CARACTERISTICILE CURBELOR PLANE

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Curbele plane clasice reprezintă un fundament al geometriei analitice şi al aplicaşiilor în fizică, inginerie şi grafică computerizată. Articolul îşi propune să ofere un suport didactic compact, dar riguros, pentru studenţii şi profesorii de matematică (nivel liceal - universitar), reunind treisprezece curbe plane fundamentale din geometria analitică şi motivarea utilităţii curbelor plane în ştiinţele exacte şi tehnice (fizica mişcării, proiectarea mecanismelor, grafică computaţională). Fiecare curbă este însoţită de:

- ecuaţii (carteziană, parametrică şi, acolo unde este relevant, polară);
- un set minimal de parametri numerici pentru exemplificare;
- un grafic clar, plasat într-un sistem de coordonate cu axele şi grilă, pentru evidenţierea formei şi a simetriilor.

Curbele analizate demonstrează bogăţia formelor geometrice ce pot fi descrise prin ecuaţii relativ simple. Corelarea formulelor analitice cu reprezentările

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grafice oferă o imagine completă asupra proprietăţilor lor metrice şi topologice, constituind un instrument didactic eficient pentru studiul geometriei analitice. Prin combinarea explicaţiilor analitice cu reprezentarea vizuală, documentul creează un cadru coerent pentru înţelegerea "semnăturilor" matematice ale fiecărei curbe. Această abordare:

1. consolidează intuiţia geometrică;
2. pregăteşte terenul pentru aplicaţii pragmatice (proiectarea de profile mecanice, simularea traiectoriilor, generarea de pattern-uri grafice);
3. familiarizează cititorul cu trecerea între diferite sisteme de coordonate şi tehnici de parametrizare - aptitudini esenţiale în matematica aplicată contemporană.

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METODA COEFICIENŢILOR NEDETERMINAŢI ÎN INTEGRAREA FUNCŢIILOR RAŢIONALE

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În procesul de integrare a funcţiilor raţionale, şi nu numai, se utilizează metoda coeficienţilor nedeterminaţi, ce de regulă, presupune obţinerea şi rezolvarea unui sistem de n ecuaţii liniare ($n \geq 2$, $n \in \mathbf{N}$).

În cazul unui număr de coeficienţi nedeterminaţi mai mare decât trei, această metodă necesită calcule destul de anevoioase ([1, 2]). Se propune o altă metodă ([3, 4, 5]) pentru a determina fără eforturi deosebite coeficienţii nedeterminaţi, ce presupune utilizarea noţiunilor de rădăcină multiplă şi proprietăţile ei, numere complexe etc.

Descompunerea unei fracţii raţionale în sumă de fracţii raţionale simple se utilizează pe larg în mai multe compartimente ale matematicii superioare de la facultate, ca de exemplu, în serii numerice, ecuaţii diferenţiale, calcul operaţional, teoria funcţiilor de o variabilă complexă etc.

[1] *Speaking author*: I. Jardan

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APLICAREA INTELIGENŢEI ARTIFICIALE ÎN MONITORIZAREA CURRICULUMULUI ŞCOLAR

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Educaţia contemporană se află într-un proces continuu de transformare, impulsionată de dinamica accelerată a schimbărilor sociale, tehnologice şi economice. În acest context, curriculumul şcolar necesită flexibilitate, adaptabilitate şi actualizare constantă pentru a corespunde cerinşelor unei societăţi în plină evoluţie. Metodele tradiţionale de evaluare a eficienţei curriculare, precum testele standardizate sau interviurile, sunt tot mai limitate în faţa noilor cerinţe de analiză rapidă şi complexă.

Inteligenţa artificială (IA) oferă soluţii moderne pentru analiza şi optimizarea curriculumului, prin capacitatea de a procesa volume mari de date, de a identifica tipare şi de a oferi feedback automat. Aceasta permite monitorizarea funcţionării curriculumului, identificarea problemelor, personalizarea învăţării şi sprijinirea reformelor educaţionale. Prin integrarea datelor din diverse surse, IA oferă o imagine clară şi în timp real asupra procesului educaţional.

Prin colectarea şi integrarea datelor educaţionale multimodale (din platforme de învăţare, evaluări, activităţi extracurriculare şi feedbackul elevilor), IA permite o analiză holistică şi în timp real a implementării curriculumului. Modelele predictive pot evalua funcţionalitatea secvenţelor curriculare, identificând zonele ineficiente şi propunând ajustări bazate pe date obiective. Totodată, IA sprijină personalizarea educaţiei prin adaptarea conţinuturilor la nevoile individuale, contribuind la reducerea abandonului şcolar şi la optimizarea progresului. Sistemele analitice inteligente furnizează rapoarte interactive factorilor decizionali, facilitând intervenţii rapide şi fundamentate în politicile curriculare.

Astfel, IA redefineşte paradigma evaluării curriculumului, oferind oportunităţi pentru o educaţie adaptivă şi centrată pe date. Pentru a valorifica acest potenţial, este esenţială colaborarea între cercetători, decidenţi, dezvoltatori de tehnologie şi cadre didactice.

În concluzie, integrarea IA în monitorizarea curriculumului şcolar promite o ameliorare semnificativă a calităţii educaţiei. Prin analiza extinsă a datelor, depistarea carenţelor, personalizarea parcursurilor de învăţare şi oferirea de feedback instantaneu, IA are potenţialul de a revoluţiona metodele de evaluare şi adaptare a programelor educaţionale. Esenţială pentru valorificarea acestui potenţial este o abordare responsabilă a provocărilor etice şi practice, astfel încât tehnologia să sprijine efectiv cadrele didactice şi să contribuie la un sistem educaţional mai performant şi echitabil. Cercetările viitoare ar trebui să se axeze pe evaluarea impactului pe termen lung al IA asupra echităţii în educaţie şi pe conturarea unor modele de integrare etică şi sustenabilă a acesteia în politicile curriculare.

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THE USE OF ARTIFICIAL INTELLIGENCE IN ESSAY-TYPE QUESTION ASSESSMENT

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Current trends show that the use of artificial intelligence (AI) in educational technologies is becoming increasingly widespread, as in other areas of life. The use of AI in education is encouraged both among students [1] and teachers [2]. For example, teachers use large language models (LLMs) to create educational materials, correct errors in texts, and generate questions based on educational content. Of particular interest is the use of LLMs for assessing individual student work, such as essays, reports, laboratory reports, etc. Moreover, among all types of questions found in tests, only essay-type questions require manual grading, which makes them the most labor-intensive for teachers and the most subjective to assess. Therefore, the use of AI to automate the assessment of essay-type questions is an important and relevant challenge.

This article proposes a model for the assessment of essay-type questions using LLMs.

For accurate assessment of essays with LLMs, the following aspects should be considered:

1. **Context**: LLMs should be provided with a context that matches the theme of the essay.

2. **Role**: LLMs should be configured to perform the role of an assessor.

3. **Assessment criteria**: Clear criteria for essay assessment should be defined, such as argumentation, originality, grammar, and style.

4. **Feedback**: LLMs can provide feedback on essays, indicating strengths and weaknesses, which helps students improve their writing skills.

5. **Ethics and transparency**: It is important to ensure the ethical use of LLMs, including transparency in how they make decisions and assess essays.

6. **Adaptability**: LLMs can adapt to different writing styles and student proficiency levels.

7. **Integration with educational platforms**: LLMs must be integrated into existing educational platforms, simplifying the assessment process and making it more accessible for teachers and students.

The conclusions of LLMs can be used to automate the assessment of essay-type questions, which helps reduce grading time and increase objectivity. However, it is important to remember that LLMs cannot fully replace human assessors, especially in cases requiring deep understanding of context and nuances. Therefore, the use of LLMs should complement traditional assessment methods; their conclusions are advisory and should be verified by a teacher.

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ABOUT A STATISTICAL PROBLEM

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A problem with applications in statistics is proposed for solution, for which methodological aspects for mathematical solution are presented.

Then, the solution obtained by implementing a program in the R programming language, one of the most current programs used in statistics, is presented.

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FOSTERING CRITICAL AND CREATIVE THINKING IN AN UNDERGRADUATE MATHEMATICAL ANALYSIS COURSE THROUGH REFLECTIVE PRACTICE AND COMPLEX TASKS

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In the Republic of Moldova, mathematics and teaching careers struggle to attract high school graduates, resulting in small cohorts of university students pursuing mathematics. Many first-year undergraduates enter university with significant knowledge gaps and a tendency toward superficial learning, which hampers their progress in rigorous mathematics courses. This situation, observed for instance at Alecu Russo Balti State University, highlights the need for innovative pedagogical strategies to re-engage students and rebuild their foundational skills. The study was conducted with first and second year students enrolled in the Bachelor's program in Mathematics and Informatics, aimed to prepare future secondary school mathematics teachers.

The rise of artificial intelligence (AI) tools in education adds complexity to this challenge. Students increasingly rely on automated problem solvers and AI-driven platforms for quick answers, often without engaging in critical thinking or deep reasoning. While such tools can support learning, overuse may hinder the development of independent and creative thought. This trend highlights the

need to cultivate higher-order thinking skills, enabling students to use AI as a support rather than a substitute. Our work aims to strengthen these cognitive capacities as a basis for both academic growth and responsible professional conduct.

In response to these challenges, our study explores a pedagogical model that integrates reflective practice and complex, cognitively demanding tasks into undergraduate Mathematical Analysis courses. The goal is to engage students more deeply in the learning process by encouraging them to monitor their reasoning, confront misconceptions, and apply theoretical concepts in novel situations. Reflective tasks help students analyze their problem-solving approaches and understand the roots of their mistakes, while complex tasks are designed to stimulate creativity, logic, and independent thinking beyond routine procedures.

Our instructional model combines two main components. First, structured metacognitive reflection sheets were administered after written assessments, prompting students to analyze their problem-solving strategies, identify the nature of their mistakes, and outline improvement plans. Second, specially designed tasks requiring non-algorithmic thinking were introduced. These included open-ended problems, unfamiliar formats, and challenges that required reasoning across multiple representations. Students were encouraged to articulate their thought processes, recognize patterns in their errors, and adjust their strategies. This dual approach sought to break passive learning habits and foster intellectual autonomy.

Together, these strategies form a coherent instructional model that blends traditional teaching with independent learning. Core content is delivered through interactive lectures, while students reinforce knowledge at their own pace via supplementary materials and exercises on the Moodle platform. Metacognitive reflection sheets bridge the gap between personal study and classroom dialogue, prompting learners to connect theory with practice and arrive prepared with deeper questions. This integrated approach fosters a reflective and adaptive learning environment, equipping students with critical thinking, creative problem-solving, and flexible learning strategies for the AI-driven future.

Preliminary observations suggest that this approach can foster more meaningful engagement with mathematical content, improve error awareness, and support the gradual development of independent reasoning. Although the implementation is still in its early stages, results indicate promising directions for teaching Mathematical Analysis in contexts where motivation and preparedness are limited. We argue that a combined use of reflective practice and complex tasks may serve as a viable strategy to cultivate critical and creative thinking in higher mathematics education.

SOME APPLICATIONS WITH POINTS AND LINES IN THREE-DIMENSIONAL EUCLIDEAN SPACE SOLVED IN MAPLE

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A method for approaching some analytical geometry problems using the Maple environment, related to some applications with points and lines in the three-dimensional Euclidean space, problems that classically solved involve the application of the same formula several times and a lot of time would be lost is presented. First, the classical solution method is presented, using the application of the formulas presented in the Analytical and Computational Geometry courses [3], then, the same problems are solved using the Maple environment [1-2].

In this paper is presented a faster solving method that involves the Maple environment to determine:

a) the equations of a straight line when two points are known;

b) the relative positions of two lines (parallel, overlapping, perpendicular, secant);

c) the coordinates of the point of intersection of two secant lines;

d) the measure of the angle between two secant lines;

e) the distance between two points;

f) the distance between two parallel lines;

g) the distance from a point to a line on which the point does not lie.

Finally, the graphical representation of the points in space is made, as well as the lines determined by them.

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PROBLEME DE GEOMETRIE COMPUTAŢIONALĂ ÎN CONTEXTUL COMPETIŢIILOR DE PROGRAMARE

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Competiţiile de programare sunt un subiect foarte cunoscut şi discutat la etapa actuală. Relevanţa lor nu mai trebuie demonstrată, fiind susţinută de extinderea accelerată a digitalizării în toate domeniile activităţii umane. Tematica problemelor incluse în calitate de probe de concurs se extinde tot mai mult cuprinzând diverse domenii. Analiza comparativă realizată de profesorul Anatol Gremalschi între IOI Syllabus şi Curriculumul şcolar de Informatică din Republica Moldova relevă faptul că Olimpiada Internaţională de Informatică (IOI) se desfăşoară în baza unui curriculum aprobat anual de Comitetul Ştiinţific Internaţional. Acest document defineşte explicit domeniile de competenţă necesare participanţilor, structurate în patru arii fundamentale: matematică, informatică, inginerie software şi alfabetizare digitală.

Astfel, în setul de probleme propuse competitorilor se regăseşte, cel puţin, o problemă de geometrie computaţională. Rezolvarea acestui tip de probleme solicită de la competitori cunoştinţe de geometrie începând cu spaţii unu, doi, trei-dimensionale; ecuaţiile dreptei, cercului, elipsei, intersecţia a două drepte, intersecţia a două figuri în plan, algoritmi de calcul a înfăşurătoarei convexe, apartenenţa la un poligon, nucleul poligonului etc.

Intersecţia acestui tip de probleme cu tehnicile de programare măreşte considerabil complexitatea/ dificultatea acestor probleme făcând să fie utilizate de juriu pentru probele de departajare.

Din analiza rezultatelor elevilor la Olimpiada Republicană de Informatică pentru ultimii 10 ani, se poate clar evidenţia faptul că, elevii olimpici întâlnesc dificultăţi în rezolvarea acestui tip de probleme. O explicaţie în acest sens ar fi pregătirea insuficientă a elevilor referitor la acest tip de probleme, inclusiv şi deţinerea de către elevi a unor competenţe matematice mai limitate.

PROBLEMATIZAREA – CHEIE PENTRU O LECŢIE ALTFEL

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Curriculum Naţional la Matematică, 2019 susţine că indiferent de tip, lecţia de Matematică, trebuie să fie o lecţie modernă, axată pe obiective, pe formarea competenţelor şi să fie centrată pe elevi. Adică "activitatea profesorului în cadrul lecţiei constituie de regulă 30%, iar activitatea elevilor – 70% din timpul ei". Tot curriculum afirmă că "predarea - învăţarea materiei noi se face de regulă, prin crearea situaţiei-problemă". De aici şi conchidem importanţa aplicării problematizării la orele de matematică.

Metoda problematizării este o metodă complexă, dar extrem de valoroasă. Complexitatea sa vine din: diversitatea formelor de aplicare, cerinţelor ridicate pentru profesor (gândire pedagogică, flexibilitate), cât şi nivelul de implicare intelectuală a elevilor, dar tocmai aceste lucruri o fac eficientă în dezvoltarea gândirii critice şi autonome. În continuare sunt prezentate două exemple de aplicare a problematizării la orele de matematică, prima presupune demonstrarea unei formule matematice, iar a doua conţine probleme din cotidian.

În anul de studii 2024-2025, la disciplina Matematica, în clasa a 9-a, la capitolul "Poliedre", ne-am propus o activitate, care face conexiune între tema 65 "Volumul prismei drepte" şi 69 "Volumul piramidei regulate (triunghiulare, patrulatere, hexagonale)". În urma observaţiei şi a analizei formulelor celor două corpuri, elevii claselor a 9-a au observat că volumul prismei diferă de volumul piramidei prin simplul fapt că în produsul dintre aria bazei şi înălţimea a apărut un factor nou "1/3". De aici şi a apărut întrebarea elevilor, ce este cu această treime din volumul piramidei. Împreună cu ei au fost efectuate transformări, şi s-a ajuns la concluzia că: $V_{prisma} = 3 \cdot V_{piramida}$. Această relaţie a fost luată drept provocare. Clasa formată din 30 elevi a fost împărţită în grupuri a câte 5 elevi. Grupurile au fost dotate cu următoarele materiale: plastilină şi scobitori din lemn de diferite lungimi. La dispoziţie elevii au avut 10 minute pentru a construi trei piramide. Pasul următor a fost să le unească între ele. Produsele grupurilor au fost diferite, procesul a implicat eforturi considerabile, a fost nevoie de răbdare şi perseverenţă pentru a ajunge în final la o prismă, care este formată din trei piramide.

La tema "Aria şi volumul piramidei regulate" elevilor din clasa a 9-a le-a fost propus un set de probleme din cotidian cu tematica "Muzeului Luvru din Paris", "Pyraminx-ul", "Punguţe pentru cadou", "Plicul standard de ceai", "Piramida orgon", "Cercel în formă de piramidă patrulateră regulată", "Piramida

lui Keops (Khufu)". Un indicator al succesului aplicării acestor activităţi, probleme, situaţii - probleme din cotidian la elevii claselor a 9-a, ar fi punctajul obţinut la Pretestarea la Matematică 2025, la itemul zece. Din cei 61 de elevi, care au fost actori principali în cadrul lecţiilor altfel, 55,73% (34 de elevi) au acumulat 4 puncte, 9,83% (6 elevi) au acumulat 3 puncte, 8,19% (5 elevi) au acumulat 2 puncte, 9,83% (6 elevi) au acumulat un punct şi 16,39%(10 elevi) au acumulat zero puncte.

În concluzie, putem afirma că prin utilizarea metodei problematizării, lecţiile de matematică devin mai dinamice, mai eficiente şi contribuie la formarea unor competenţe esenţiale pentru succesul elevilor atât în şcoală, cât şi în viaţa de zi cu zi.

POTENŢIALUL E-TESTĂRII PE MOODLE PENTRU STUDIUL MATEMATICII ŞI INFORMATICII

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e-Testingul pe Moodle facilitează (auto) instruirea, (auto) treningul, (auto) evaluarea continuă formativă, intermediară şi finală; permite păstrarea unei motivaţii înalte şi a unei atitudini pozitive a respondenţilor; permite eliminarea rutinei şi o economie colosală a timpului profesorilor, necesare pentru evaluare, lăsându-i să se concentreze pe aspectele pedagogice şi ştiinţifice; sporeşte conformitatea dintre curriculumul planificat - predat - evaluat prin acoperirea întregii materii; asigură transparenţa, uniformitatea, unicitatea, obiectivitate maximă, flexibilitate înaltă, feedback instant, înregistrarea automată a rezultatelor, standardizarea şi îmbunătăţirea continuă a itemilor şi feedbackului şi altele.

Un test bun este aplicabil, fidel, obiectiv şi valid - adică uşor de administrat şi interpretat, produce rezultate constante, permite evaluări consecvente şi măsoară corect ceea ce îşi propune [1]. Pe de altă parte, un test fidel, valid, comparabil, obiectiv, aplicabil etc. ar trebui să fie şi rapid, şi precis, şi ieftin. Pentru atingerea acestui scop, colecţiile de itemi, bateriile de teste şi testele de autor generate în baza lor ar trebui să fie bine planificate, specificate, administrate etc. [2]. Acest lucru este posibil doar în condiţii de automatizare a proceselor de planificare-dezvoltare a colecţiilor de itemi şi bateriilor de teste, de corectare şi feedback [3].

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Ultimele versiuni Moodle permit crearea de itemi parametrizaţi variabili, ideali pentru studiul matematicii (calcule simple, formule, plugin STACK MAXIMA [4]) şi informaticii / programării (CodeRunner). Experimentarea unor asemenea tipuri de itemi variabili a fost realizată pe platformele de eLearning ale USM https://moodle.usm.md şi CEITI https://moodle1.ceiti.md/login/index.php/ în cadrul Proiectului de inovare şi transfer tehnologic "Creşterea performanţei academice la disciplina Informatica/TIC din contul implementării unor instrumente standardizate de evaluare pe platforma de eLearning Moodle", aprobat prin ordinul MEC nr.364 din 18.03.2025.

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ÎNVĂȚAREA MATEMATICII PRIN ITEMI PARAMETRIZAȚI

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Digitalizarea schimbă rapid modul în care se desfășoară procesul educațional. Itemii parametrizați devin un instrument util în acest context, oferind exerciții unice și personalizate eficiente de învățare și evaluare. Pentru matematică, acești itemi contribuie la înțelegerea mai profundă a noțiunilor și la îmbunătățirea continuă a abilităților.

Platforma Moodle, prin opțiunea de creare a itemilor de tip calculat, numeric, formulă oferă un cadru ideal pentru dezvoltarea evaluării formative și sumative, adaptate la nivelul fiecărui elev.

Acești itemii permit generarea automată de exerciții matematice cu valori variabile, ceea ce asigură unicitatea fiecărui test și stimulează analiza logică,

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raționamentul matematic și autonomia în învățare. Astfel, elevii nu doar repetă formule, ci învață să aplice conceptele în contexte variate.

Acest tip de evaluare transformă testarea într-un instrument activ de învățare, oferind feedback automat instant și contribuind la dezvoltarea competențelor matematice prin exercițiu constant și personalizat. În același timp, profesorii beneficiază de un sistem eficient, care reduce timpul necesar pentru corectare și permite o monitorizare clară a progresului elevilor.

Un test matematic bine conceput pe Moodle în baza itemilor de tip calculat, numeric, formulărespectă criteriile fundamentale de calitate a evaluării (Aplicabilitate, Fidelitate, Obiectivitate, Validitate etc.[1]), este unic, standardizat, poate fi aplicat repetat în toate tipurile de evaluare, de către toate instituțiile ce aplică același curriculum, fără riscul de copiere sau fraudare a examenelor.

Această abordare a fost testată și validată cu succes în cadrul platformelor Moodle USM și CEITI, demonstrând impact pozitiv asupra învățării matematice în Proiectul "Creșterea performanței academice la disciplina Informatica/TIC prin implementarea unor instrumente standardizate de evaluare pe platforma de eLearning Moodle", aprobat prin ordinul MEC nr. 364 din 18.03.2025.

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BENEFITS OF USING DIGITAL APPLICATIONS IN THE STUDY OF MATHEMATICS

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This study presents the benefits of using the GeoGebra application in the study of mathematics, regardless of the branch - algebra, geometry, mathematical analysis, statistics - at school, during the training activity or at home, during individual study. It offers the possibility of obtaining precise geometric figures in a relatively short time, can be seen as a self-assessment tool for those who solve problems individually, ensures a fair ratio between scientific rigor and accessibility, contributes to recording progress in the mathematical discipline for students who use it, stimulating the level of concentration during the time dedicated to study.

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SETAREA ȘI UTILIZAREA OPȚIUNILOR ÎN APLICAȚIA "EXPLORINGBMP"

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Aplicația ExploringBMP.exe destinată explorării fișierelor de tip BMP, a fost elaborată în colaborare cu S. Pereteatcu [1, 2]. Aceasta oferă încărcarea și vizualizarea codului hexazecimal al conținutului fișierului BMP indicat; diferite experimente cu parametrii acestuia. În continuare vom descrie setarea opțiunilor aplicației "ExploringBMP". La selectarea butonului "Settings", apare caseta modală de dialog cu același nume, care conține două grupuri de casete de selectare independente: "Requirements", "AntiSpyware"; grupuri de butoane radio care se exclud reciproc "How to write text in file?"; două elemente NumericUpDown pentru specificarea informațiilor numerice: "AutomaticDelay value", "When the ProgressBar"; două butoane: "Save", "Load". Indicatoarele de grup "Requirements" determină ce secțiuni ale codului curent în format BMP trebuie verificate pentru corectitudine.

Caseta "All", dacă nu este bifată, setarea acesteia va face ca toate celelalte casete de selectare din acest grup să fie bifate. Dacă este bifată și, prin urmare, toate celelalte casete ale acestui grup sunt bifate, atunci resetarea ei provoacă resetarea tuturor celorlalte casete ale acestui grup. Dacă toate casetele de selectare din acest grup sunt bifate, atunci debifarea oricărei casete de selectare, alta decât "All", va reseta caseta de selectare "All".

Casetele din grupul "AntiSpyware" determină ce secțiuni de cod în format BMP trebuie curățate atunci când selectăm opțiunea "Clean Up Spyware" din meniul contextual principal.

Grupul de butoane radio "How to write text in file?" determină cum va arăta conținutul fișierului text generat prin selectarea opțiunii "Print to text file". Folosind opțiunea "AutomaticDelay value", putem modifica valoarea parametrului AutomaticDelay. Folosind opțiunea "When the ProgressBar", vedem indicat numărul minim de celule de octeți care trebuie procesate în orice proces iterativ. De exemplu, ascunderea spațiului nefolosit.

Butoanele "Save" și "Load" ne permit, respectiv, să salvăm configurația valorilor curente ale atributelor în fișierul text specificat și să extragem din fișierul text specificat.

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STATISTICAL METHODS USED IN INSURANCE

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Actuarial science appeared from the 18th century through a combination of the interest rate with mortality tables. The accurate calculation of life insurance premiums is a foundational task within actuarial science, directly influencing the financial stability of insurance providers and the fairness of policy pricing. This process is underpinned by several core elements, including mortality tables [1], demographic characteristics of the insured population (such as age and sex), general health trends, contract duration, and the level of insured sums. Actuaries rely heavily on mortality tables, which provide statistical estimates of death probabilities across age groups and are essential in assessing life expectancy [2] and pricing risk [4]. Demographic factors such as increasing age and male sex are associated with elevated mortality risks, necessitating risk-adjusted premium levels [3].

The general health of the population, often reflected in public health data and morbidity rates, further influences projected mortality, affecting future liabilities and premium assumptions. The contract period is also a significant determinant: longer-term contracts or permanent insurance (e.g., whole life) present increased uncertainty and necessitate higher reserve requirements [6]. The sum insured — the monetary value guaranteed upon the insured event — directly scales the insurer's risk exposure, thereby influencing premium magnitude. Several types of life insurance policies exist, each with unique implications for premium calculation. Survival insurance provides benefits only if the insured survives beyond the term, while whole life insurance ensures a payout upon death at any time, requiring more robust long-term risk forecasting. Mixed life insurance combines death and survival benefits, demanding a hybrid actuarial approach [5].

In sum, the actuary's role in premium calculation is multifaceted, integrating statistical modeling, financial theory, and population health data to create sustainable and equitable insurance products. Their expertise ensures a balance between risk management and affordability, contributing to the long-term viability of life insurance systems.

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APLICAREA SOFTWARE-ULUI SPECIALIZAT ÎN ANALIZA CALITATIVĂ A DATELOR

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Analiza calitativă a datelor este o componentă esențială a cercetării științifice din domeniul social, facilitând explorarea în profunzime a fenomenelor sociale. Spre deosebire de analiza cantitativă, axată pe date numerice și inferențe statistice, analiza calitativă urmărește reformularea, explicarea sau teoretizarea unei mărturii, a unei experienţe sau unui fenomen, logica căreia este descoperirea sau constituirea sensului. Acest proces, de regulă complex și iterativ, presupune o serie de activități, precum transcrierea, codarea, categorizarea și interpretarea datelor. Având în vedere creșterea constantă a volumului de informații colectate, utilizarea software-ului specializat nu mai reprezintă doar un avantaj, ci o necesitate pentru gestionarea eficientă și analiza aprofundată a datelor calitative.

Programele dedicate, precum NVivo, ATLAS.ti, MAXQDA, NUD*IST, The Ethnograph, Dedoose sau QDA Miner, oferă funcționalități esențiale care susțin întregul parcurs analitic. Acestea permit importul și organizarea datelor diverse (text, audio, video), codarea (deductivă/inductivă) segmentelor, adăugarea de memo-uri și adnotări, și interogarea/regăsirea rapidă a informațiilor. De asemenea, facilitează vizualizarea și modelarea relațiilor dintre concepte (hărți conceptuale, rețele) și includ funcții de analiză de text avansată (frecvență, concordanță). Nu în ultimul rând, oferă opțiuni de export și raportare a rezultatelor.

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Software-ul specializat dedicat analizei calitative a devenit un instrument indispensabil în cercetarea contemporană. Prin facilitarea procesării datelor calitative (organizare și codare) și interpretarea acestora, acestea contribuie la asigurarea acurateței datelor și creșterea rigorii metodologice în procesul de lucru cu datele.

Alegerea unui astfel de instrument trebuie să fie ghidată de obiectivele cercetării, tipul datelor analizate și competențele tehnice ale echipei. Pe măsură ce tehnologia avansează, integrarea inteligenței artificiale promite să extindă capacitățile acestor platforme, accelerând analiza și oferind perspective mai profunde asupra datelor calitative. Cu toate acestea, rolul cercetătorului rămâne esențial în interpretarea semnificațiilor și în asigurarea validității datelor.

REALIZAREA APLICAȚIILOR DE SUPORT PENTRU CURSUL "GRAFICĂ 2D PE CALCULATOR"

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În susținerea cursului de grafică 2D pe calculator au fost elaborate 8 aplicații [1] de tip GUI în limbajul C#, folosind biblioteca de clase .NET Framework.

Aplicația RGB-CMY-CMYK.exe demonstrează structura modelului de culori aditiv RGB, structura modelului de culori substractiv CMY, legătura dintre ele, trecerea de la modelul CMY la modelul CMYK și invers, sensul canalului de transparență și altele.

Aplicația GrMycos.exe permite desenarea graficului funcției cos(x) și diferite experimente cu intervalul, cu precizia, cu numărul de puncte intermediare și altele.

Aplicația Diagram.exe demonstrează construirea diagramelor prin diferite forme (indicii absoluți și relativi) cu utilizarea funțiilor mai avansate de desenare.

Aplicațiile Bezier3.exe și Bezier4.exe permit editarea curbelor Bezier respectiv de grad 3 și de grad 4. Ele asigură posibilitatea de a experimenta atât cu coordonatele punctelor de control, cât și cu coeficienții polinoamelor caracteristici, cu desenarea graficelor lor.

Aplicația Bezier-n.exe asigură editarea curbelor Bezier de grad n cu scopul de a se convinge în imposibilitatea utilizării curbelor Bezier de graduri mai mari de 3 în editoare grafice.

Aplicația Bezier3-Lant.exe demonstrează editarea lanțului din două curbe Bezier de grad 3, dând posibilitatea de a experimenta cu puncte de control și cu modul de conectare a curbelor (Cusp, Smooth, Symmetric).

Aplicația ExploringBMP.exe - explorarea fișierelor .BMP, a fost elaborată în colaborare cu Marin Gh. Ea oferă încărcarea și vizualizarea codului hexazecimal al conținutului fișierului BMP indicat; diferite experimente cu parametrii acestuia.

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MODELING OF THE FILAMENT NUCLEATION-ELONGATION PROCESS ON DAMAGED DNA

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The paper proposes a computational model of the process of nucleation and elongation of RecA filaments, an essential component of the SOS response triggered by the bacterium Escherichia coli under conditions of genotoxic stress. The modeling is based on real experimental parameters, extracted from the specialized scientific literature, and focuses on the interaction between proteins involved in DNA repair and the dynamics of filament formation on damaged DNA regions.

Under conditions of severe stress, such as exposure to UV radiation or chemical agents (e.g., mitomycin C or alkylating agents), the cell activates DNA repair mechanisms. These aim to correct single-strand breaks (SSB) and double-strand breaks (DSB). A crucial step in this process is the formation of RecA nucleoprotein filaments, which bind to single-stranded DNA regions and initiate the activation of the SOS gene expression regulatory cascade.

The model is implemented using Timed Hybrid Petri Nets, which allow the simultaneous description of discrete (molecular events) and continuous (protein concentrations) dynamics. A central component of this model is the detailed simulation of the process of nucleation and elongation of RecA filaments, closely related to the induction of regulatory gene expression.

The results of the simulations illustrate the temporal variations of lexA and recA gene expression, as well as the concentrations of key proteins involved. The time evolution of the processes of formation and dissociation of nucleoprotein filaments on damaged DNA is also illustrated. These results correspond to known experimental data, thus validating the predictive power of the proposed model.

The model is also analyzed in the Mathematical Modeling in Bioinformatics course for master's students, having implications for areas such as self-organization of complex biological systems, molecular computing, membrane computing, and hybrid systems. By integrating theoretical aspects and computational applications, this model provides a robust framework for understanding the mechanisms regulating the cellular response to genotoxic stress.

CURRENT ISSUES AND CHALLENGES IN THE EDUCATION OF MATHEMATICALLY GIFTED STUDENTS

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This investigation seeks to highlight the current challenges in educating mathematically gifted students, including inadequate curriculum differentiation, inconsistent identification methods, insufficient teacher training, equity concerns, underfunding of programs for these children, and limited psychodidactic research [2]. Students with advanced mathematical skills are not sufficiently challenged by the standard curriculum. For gifted in mathematics students, curricular enrichment and acceleration programs that offer advanced, research-based content, flexible pacing, opportunities for collaboration and cooperation in problem-solving, access to mentors, and competitions are effective. Participation in competitions provides students with a unique opportunity to develop advanced problem-solving skills, gain experience in scholarly mathematical activity, build confidence, and connect with a community of peers with shared interests. Equity issues arise when special programs or acceleration options are not accessible to all gifted students. There are ongoing debates about the best ways to group students with exceptional mathematical skills: homogeneous groups can provide an appropriate challenge but may negatively affect them emotionally, while heterogeneous groups often fail to meet advanced needs. Studies that aim at equity from the perspective of gender belonging are necessary.

The results of investigations conducted between 2010 and 2012 regarding the preparation of teachers for working with gifted in mathematics are reflected in [1]. Recent research and the application of specific questionnaires among mathematics teachers regarding extracurricular activities and the specifics of working with gifted students allow us to observe a different situation than what was 15

[1] *Speaking author*: L. Sali

years ago. Initial and continuous teacher training programs do not prioritize the education of gifted students, which leads to a limited capacity to provide appropriate cognitive challenges. We observe a lack of systematic empirical research that would provide factual material for appropriate theoretical investigations. More research is needed to develop and validate theoretical models, best practices, and innovative approaches that stimulate students' creativity and mathematical potential in an environment that combines: intellectual development, personal development, and communication skills development [3, 4].

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INDIVIDUALIZATION OF MATHEMATICS EDUCATION IN UNIVERSITY EDUCATION: PROBLEMS AND TECHNOLOGIES

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Modern university mathematics education faces the need to adapt to different levels of students' preparedness, motivation and cognitive styles. In the conditions of digitalization and mass education, individualization becomes the most important direction of didactic development.

One of the most important directions is the implementation of differentiated educational trajectories, which presuppose the existence of basic and advanced levels for the study of mathematical subjects. This model makes it possible to take into account students' interests and actual level of preparation, allowing them to choose the pace and degree of difficulty of the content studied.

Adaptive learning platforms (e.g. those with artificial intelligence elements) are an effective personalization tool. They provide tasks with automated feedback, adapt according to students' level of knowledge, identify errors and propose individualized learning paths. In this way, they contribute to the active involvement of students and the formation of a conscious cognitive approach.

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Equally important is the development of the tutoring and mentoring system, in which the teacher or an advanced student provides support in choosing the optimal educational trajectory, stimulates motivation and offers individual advice. Effective formats include small groups, regular meetings or online tutoring through educational platforms.

However, the implementation of individualization entails a number of challenges: limited resources (time, teachers' workload), the need for digital competences, the difficulty of balancing individual and collective learning. These issues require a systemic approach, combining didactic principles with technological and organizational solutions. Thus, individualization of university mathematics education is not only a modern trend, but also an effective tool for improving the quality of the educational process. It is essential to provide methodological and technological support for teachers, develop digital infrastructure and apply flexible teaching models.

RETRAINING IN MATHEMATICS: BETWEEN NECESSITY AND EFFICIENCY - SOLUTIONS FOR OPTIMIZING THE IMPACT ON THE EDUCATIONAL SYSTEM

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The retraining program in Mathematics was introduced as a necessity to address the shortage of qualified teaching staff in the Republic of Moldova. Although the program plays a crucial role in supporting the educational system, its real efficiency in training professional teachers leaves much to be desired. Key challenges include the limited duration of the program and its predominantly online format, which fail to cover gaps related to participants' insufficient digital skills. However, a positive aspect of the program is the requirement to obtain the first mandatory bachelor's degree, an essential element in ensuring a minimum level of theoretical preparation. In this context, we propose a set of efficiency-enhancing tools aimed at improving the impact of the retraining process. These tools include continuous mentoring, the organization of hybrid workshops with practical applicability, and a strong focus on developing digital skills, all with the goal of facilitating the rapid and sustainable integration of newly qualified teachers into the educational system.

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ROLUL LIMBAJELOR FORMALE ÎN DEZVOLTAREA GÂNDIRII COMPUTAȚIONALE LA STUDENȚI

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În contextul formării viitorilor specialiști în informatică, dezvoltarea gândirii computaționale reprezintă una dintre cele mai importante mize ale învățământului superior. Gândirea computațională este mai mult decât simpla capacitate de a programa: ea implică aptitudini precum abstractizarea, recunoașterea de tipare, decompoziția problemelor, modelarea și raționamentul logic. În acest proces complex de formare, disciplina Limbaje formale și automate joacă un rol esențial, deși adesea este percepută ca fiind prea teoretică sau dificilă de aplicat.

În realitate, limbajele formale constituie fundamentul multor componente esențiale ale informaticii moderne: limbaje de programare, compilatoare, interpretoare, recunoașterea de tipare, procesarea limbajului natural, chiar și inteligența artificială. Prin intermediul acestei discipline, studenții au ocazia să își dezvolte gândirea algoritmică într-un mod riguros și formalizat. Modelele teoretice precum automate finite, automate pushdown sau mașini Turing nu sunt doar concepte abstracte, ci oferă o metodă clară de reprezentare a proceselor computaționale. Învățând să construiască și să analizeze aceste modele, studenții deprind un tip special de raționament - unul logic, structurat și predictibil - care le permite să abordeze și să rezolve probleme complexe.

Un exemplu concret în acest sens este exercițiul de proiectare a unui automat finit determinist care recunoaște șiruri binare divizibile cu 3. Dincolo de algoritmul efectiv, studenții învață să traducă o cerință exprimată în limbaj natural într-un model matematic exact. Ei identifică stările, definesc tranzițiile și testează modelul, dezvoltând astfel o gândire sistemică și capacitatea de analiză. Un alt exemplu este construirea unei gramatici independentă de context pentru expresii aritmetice cu operații de adunare și înmulțire. Acest exercițiu stimulează înțelegerea structurilor recursive și a ierarhiei operatorilor, contribuind totodată la formarea unei viziuni de ansamblu asupra modului în care funcționează parsing-ul într-un compilator.

De asemenea, conversia între diferitele forme de reprezentare a limbajelor - de exemplu, transformarea unei expresii regulate într-un automat finit sau invers - încurajează raționamentul deductiv și înțelegerea echivalenței între modele. Aceste exerciții nu doar că întăresc conceptele teoretice, dar îi învață pe studenți cum să gândească algoritmic și riguros, să caute soluții echivalente și să verifice corectitudinea acestora.

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Un rol important în susținerea acestui proces îl au instrumentele digitale, care permit simularea vizuală a automatelor, a gramaticilor și a expresiilor regulate. Prin intermediul acestor aplicații, studenții pot observa comportamentul modelelor create și pot corecta greșelile într-un mod interactiv. Această abordare vizuală completează partea teoretică, făcând procesul de învățare mai intuitiv și mai eficient.

În concluzie, Limbaje formale și automate nu este doar o disciplină teoretică din curricula informatică, ci un veritabil atelier de formare a gândirii computaționale. Prin exerciții bine alese, metode interactive și corelarea continuă cu aplicații reale, această disciplină contribuie în mod direct la dezvoltarea capacității studenților de a înțelege, modela și rezolva probleme complexe. Este de dorit ca profesorii să valorifice întregul potențial al acestei discipline, integrând metode moderne de predare și instrumente digitale, pentru a transforma conceptele formale într-o resursă practică și durabilă în formarea viitorilor specialiști IT.

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ABOUT THE DIOPHANTINE EQUATION

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In number theory Diophantine equations of the form

$$1/w + 1/x + 1/y + 1/z = m/n$$

are studied [1,2,3,4,5]. In 2018, Thingting Bai [5], determined the solutions of the equation

$$1/w + 1/x + 1/y + 1/z = 1/2.$$

In the paper stuides the solutions of the equation

$$1/w + 1/x + 1/y + 1/z = 1/3.$$

Theorem. *If the Diophantine equation* $1/w + 1/x + 1/y + 1/z = 1/n$ *has solution, then*

$$n+1 \leq w \leq 4n,$$
$$n^2+n+1 \leq x \leq 3n^2+3n,$$
$$n^4+2n^3+2n^2+n+1 \leq y \leq 2n^4+4n^3+4n^2+2n$$
and
$$2n^4+4n^3+4n^2+2n \leq z \leq n^8+4n^7+8n^6+10n^5+9n^4+6n^3+3n^2+n.$$

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SOME APPLICATIONS OF EISENSTEIN RING PROPERTIES TO SOLVING COMPETITION PROBLEMS IN NUMBER THEORY

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This paper examines some particularities of the properties of Eisenstein's ring elements that can be applied to solving some competition problems.

Let ω be a root of the polynomial x^2+x+1. The set $\mathbf{Z}[\omega]=\{a+b\omega|a,b\in\mathbf{Z}\}$ is called the set of Eisenstein integers. The set $\mathbf{Z}[\omega]$ is a subset of the complex number $\mathbf{C}$. We can thus do arithmetic as we usually do with complex numbers. The set $\mathbf{Z}[\omega]$ is closed under addition and multiplication and ($\mathbf{Z}[\omega]$,+,*) is an Euclidean ring. This ring has properties similar to those of the ring of integers Z: divisibility, unique factorization into prime factors, Euclid's algorithm, common divisors. The norm in $Z[\omega]$ is defined as the map $\mathcal{N}:\mathbf{Z}[\omega]\to N$ given by $\mathcal{N}(a+b\omega)=a^2-ab+b^2$, where $a,b\in Z$.

The properties of the elements of the Eisenstein ring are useful in solving several problems of the following type: Let $a,b,c,...$ be natural numbers such that $P(a,b,c,...)=0$. It must be shown that the number $Q(a,b,c,...)$ is composite. The method presented here can be applied mainly when $P(a,b,c,...)$ and $Q(a,b,c,...)$ are quadratic expressions in $a,b,c,....$ The basic idea comes from the following:

Lemma. *If a,b,c,d are natural numbers with the property that $ab=cd$, then there exist natural numbers m,n,p,q such that $(n,p)=1$ and $a=mn$, $b=pq$, $c=mp$, $d=nq$.*

The lemma is valid both in the ring of integers Z and in the ring of Eisenstein.

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Exemple (IMO 2001). Let $a > b > c > d$ be natural numbers such that $a^2 + c^2 - ac = b^2 + d^2 + bd$. It must be proven that $ab + cd$ is a composite number.

Using the above lemma for the Eisenstein ring, the condition in the statement can be written as follows: $(a - c\omega)(a + c\overline{\omega}) = (b + d\omega)(b + d\overline{\omega})$. According to the lemma, there are x, y, z, t in $Z[\omega]$ such that (y, z) = 1 and $a - c\omega = xy$, $a - c\overline{\omega} = zt$, $b + d\omega = xz$, $b + d\overline{\omega} = yt$.

By transforming the expressions, we obtain $ab+cd = -\frac{1}{3}y\overline{y}\sqrt{3}i(x^2\omega - \overline{x^2}\overline{\omega})$. If we consider $ab+cd$ to be the prime, we have to analyze various options for the factors on the right side of the equality and every time we obtain contradictions.

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DESPRE PROGRESII

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Se examinează progresiile aritmetice şi geometrice în cazul când raţiile sunt la rândul lor nişte progresii aritmetice sau geometrice. Pentru fiecare din ele se determină termenul general şi suma primilor n termeni:

a. progresii aritmetice cu raţia, care la rândul său este o progresie aritmetică;
b. progresii aritmetice cu raţia, care la rândul său este o progresie geometrică;
c. progresii geometrice cu raţia, care la rândul său este o progresie aritmetică;
d. progresii geometrice cu raţia, care la rândul său este o progresie geometrică.

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